Lab Manual for Physics 205/210

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Goal

Form hypotheses about interactions and test them.

Introduction

Why does a rock fall if you drop it? The ancient Greek philosopher Aristotle theorized that it was because the rock was trying to get to its natural place, in contact with the earth. Why does a ball roll if you push it? Aristotle would say that only living things have the ability to move of their own volition, so the ball can only move if you give motion to it. Aristotle's explanations were accepted by Arabs and Europeans for two thousand years, but beginning in the Renaissance, his ideas began to be modified drastically. Today, Aristotelian physics is discussed mainly by physics teachers, who often find that their students intuitively believe the Aristotelian world-view and strongly resist the completely different version of physics that is now considered correct. It is not uncommon for a student to begin a physics exam and then pause to ask the instructor, "Do you want us to answer these questions the way you told us was true, or the way we really think it works?" The idea of this lab is to make observations of objects, mostly magnets, pushing and pulling on each other, and to figure out some of the corrections that need to be made to Aristotelian physics.

Some people might say that it's just a matter of definitions or semantics whether Aristotle is correct or not. Is Aristotle's theory even testable? One testable feature of the theory is its asymmetry. The Aristotelian description of the rock falling and the ball being pushed outlines two relationships involving four objects:

earth ———— The earth is the rock's natural place. ———— rock

hand ———— The hand gives motion to the ball. ———— ball

According to Aristotle, there are asymmetries involved in both situations.

(1) The earth's role is not interchangeable with that of the rock. The earth functions only as a place where the rock tends to go, while the rock is an object that moves from one place to another.

(2) The hand's role is not analogous to the ball's. The hand is capable of motion all by itself, but the ball can't move without receiving the ability to move from the hand.

If we do an experiment that shows these types of asymmetries, then Aristotle's theory is supported. If we find a more symmetric situation, then there's something wrong with Aristotle's theory.

Observations

The following important rules serve to keep facts separate from opinions and reduce the chances of getting a garbled copy of the data:

(1) Take your raw data in pen, directly into your lab notebook. This is what real scientists do. The point is to make sure that what you're writing down is a first-hand record, without mistakes introduced by recopying it. (If you don't have your two lab notebooks yet, staple today's raw data into your notebook when you get it.)

(2) Everybody should record their own copy of the raw data. Do not depend on a "group secretary."

(3) If you do calculations during lab, keep them on a separate page or draw a line down the page and keep calculations on one side of the line and raw data on the other. This is to distinguish facts from inferences. (I will deduct 25% from your grade if you mix calculations and raw data.)

(4) Never write numbers without units. Without

units, a number is meaningless. There is a big difference bewteen "Johnny is six" and "Johnny is six feet." (I will deduct 25% from your grade if you write numbers without units.)

Because this is the first meeting of the lab class, there is no prelab writeup due at the beginning of the class. Instead, you will discuss your results with your instructor at various points.

A Comparing magnets' strengths

To make an interesting hypothesis about what will happen in part C, the main event of the lab, you'll need to know how the top (single) and bottom (large) magnets' strengths compare. Since the large magnet is made out of six of the small magnets stuck together in a stack, it would seem logical that the large magnet would be six times stronger than the single, but in this part of the lab you're going to find out for sure.

compass

Orient your magnet this way, as if it's rolling toward the compass from the north. With no magnet nearby, the compass points to magnetic north (dashed arrow). The magnet deflects the compass to a new direction.

One way of measuring the strength of a magnet is to place the magnet to the north or south of the compass and see how much it deflects (twists) the needle of a compass. You need to test the magnets at equal distances from the compass, which will produce two different angles.[1] It's also important to get everything oriented properly, as in the figure.[2]

Make sure to take your data with the magnets far enough from the compass that the deflection angle is fairly small (say 5 to 30°). If the magnet is close enough to the compass to deflect it by a large angle, then the ratio of the angles does not accurately
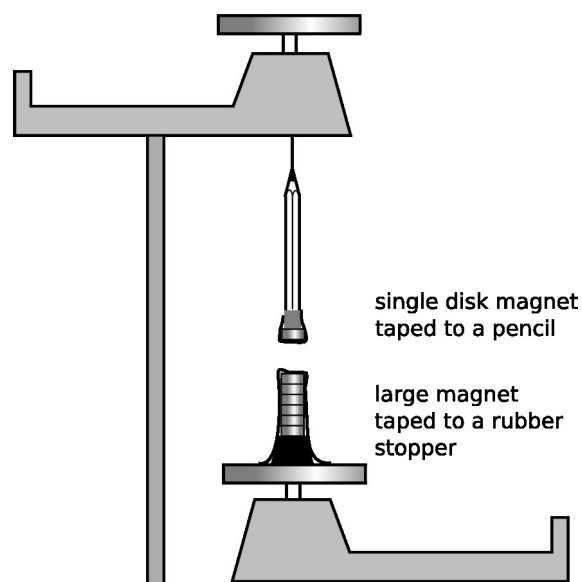
represent the ratio of the magnets' strengths. After all, just about any magnet is capable of deflecting the compass in any direction if you bring it close enough, but that doesn't mean that all magnets are equally strong.

B Qualitative observations of the interaction of two magnets

Play around with the two magnets and see how they interact with each other. Can one attract the other? Can one repel the other? Can they act on each other simultaneously? Do they need to be touching in order to do anything to each other? Can A act on B while at the same time B does not act on A at all? Can A pull B toward itself at the same time that B pushes A away? When holding one of the heavier magnets, it may be difficult to feel when there is any push or pull on it; you may wish to have one person hold the magnet with her eyes closed while the other person moves the other magnet closer and farther.

C Measurement of interactions between two magnets

Once you have your data from parts A and B, you are ready to form a hypothesis about the following situation. Suppose we set up two balances as shown in the figure. The magnets are not touching. The top magnet is hanging from a hook underneath the pan, giving the same result as if it was on top of the pan. Make sure it is hanging under the *center* of the pan. You will want to make sure the magnets are pulling on each other, not pushing each other away, so that the top magnet will stay in one place.

single disk magnet taped to a pencil

large magnet taped to a rubber stopper

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[1]There are two reasons why it wouldn't make sense to find different distances that produced the same angle. First, you don't know how the strengths of the effect falls off with distance; it's not necessarily true, for instance, that the magnetic field is half as strong at twice the distance. Second, the point of this is to help you interpret part C, and in part C, the large magnet's distance from the single magnet is the same as the single magnet's distance from the first magnet.

[2]Laying the magnet flat on the table causes the compass needle to try to tilt out of the horizontal plane, which it's not designed to do. Turning it so that it faces the compass also doesn't work, because it makes the magnet's magnetic field lie along the same north-south line as the Earth's, rather than perpendicular to it.

The balances will not show the magnets' true masses, because the magnets are exerting forces on each other. The top balance will read a higher number than it would without any magnetic forces, and the bottom balance will have a lower than normal reading. The difference between each magnet's true mass and the reading on the balance gives a measure of how strongly the magnet is being pushed or pulled by the other magnet.

How do you think the amount of pulling experienced by the two magnets will compare? In other words, which reading will change more, or will they change by the same amount? Write down a hypothesis; you'll test this hypothesis in part C of the lab. If you think the forces will be unequal predict their ratio.

Discuss with your instructor your results from parts A and B, and your hypothesis about what will happen with the two balances.

Now set up the experiment described above with two balances. Since we are interested in the changes in the scale readings caused by the magnetic forces, you will need to take a total of four scale readings: one pair with the balances separated and one pair with the magnets close together as shown in the figure above.

When the balances are together and the magnetic forces are acting, it is not possible to get both balances to reach equilibrium at the same time, because sliding the weights on one balance can cause its magnet to move up or down, tipping the other balance. Therefore, while you take a reading from one balance, you need to immobilize the other in the horizontal position by taping its tip so it points exactly at the zero mark.

You will also probably find that as you slide the weights, the pointer swings suddenly to the opposite side, but you can never get it to be stable in the middle (zero) position. Try bringing the pointer manually to the zero position and then releasing it. If it swings up, you're too low, and if it swings down, you're too high. Search for the dividing line between the too-low region and the too-high region.

If the changes in the scale readings are very small (say a few grams or less), you need to get the magnets closer together. It should be possible to get the scale readings to change by large amounts (up to 10 or 20 g).

Part C is the only part of the experiment where you will be required to analyze random errors using the techniques outlined in Appendices 2 and 3 at the back of the lab manual. Think about how you can get an estimate of the random errors in your measurements. Do you need to do multiple measurements? Discuss this with your instructor if you're uncertain.

Don't take apart your setup until lab is over, and you're completely done with your analysis — it's no fun to have to rebuild it from scratch because you made a mistake!

D Measurement of interactions involving objects in contact

You'll recall that Aristotle gave completely different interpretations for situations where one object was in contact with another, like the hand pushing the ball, and situations involving objects not in contact with each other, such as the rock falling down to the earth. Your magnets were not in contact with each other. Now suppose we try the situation shown below, with one person's hand exerting a force on the other's. All the forces involved are forces between objects in contact, although the two people's hands cannot be in direct contact because the spring scales have to be inserted to measure how strongly each person is pulling. Suppose the two people do not make any special arrangement in advance about how hard to pull. How do you think the readings on the two scales will compare? Write down a hypothesis, and discuss it with your instructor before continuing.
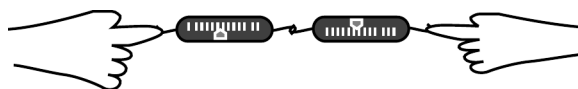
Calibrate the spring scales by sliding the metal tabs. Now carry out the measurement shown in the figure.

Self-Check

Do all your analysis in lab, including error analysis for part C. Error analysis is discussed in appendices 2 and 3; get help from your instructor if necessary.

Analysis

In your writeup, present your results from all four parts of the experiment, including error analysis for part C.

The most common mistake is to fail to address the *point* of the lab. If you feel like you don't understand why you were doing any of this, then you were missing out on your educational experience! See the back of the lab manual for the format of lab writeups.

Notes For Next Week

(1) Next week, when you turn in your writeup for this lab, you also need to turn in a prelab writeup for the next lab. on a separate piece of paper. The prelab questions are listed at the end of the description of that lab in the lab manual. Never start a lab without understanding the answers to all the prelab questions; if you turn in partial answers or answers you're unsure of, discuss the questions with your instructor or with other students to make sure you understand what's going on.

(2) You should exchange phone numbers with your lab partners for general convenience throughout the semester. You can also get each other's e-mail addresses by logging in to Spotter and clicking on "e-mail."

Rules and Organization

Collection of raw data is work you share with your lab partners. Once you're done collecting data, you need to do your own analysis. E.g., it is not okay for two people to turn in the same calculations, or on a lab requiring a graph for the whole group to make one graph and turn in copies.

You'll do some labs as formal writeups, others as informal "check-off" labs. As described in the syllabus, they're worth different numbers of points, and you have to do a certain number of each type by the end of the semester.

The format of formal lab writeups is given in appendix 1 on page 46. The raw data section must be contained in your bound lab notebook. Typically people word-process the abstract section, and any other sections that don't include much math, and stick the printout in the notebook to turn it in. The calculations and reasoning section will usually just consist of hand-written calculations you do in your lab notebook. You need two lab notebooks, because on days when you turn one in, you need your other one to take raw data in for the next lab. You may find it convenient to leave one or both of your notebooks in the cupboard at your lab bench whenever you don't need to have them at home to work on; this eliminates the problem of forgetting to bring your notebook to school.

For a check-off lab, the main thing I'll pay attention to is your abstract. The rest of your work for a check-off lab can be informal, and I may not ask to see it unless I think there's a problem after reading your abstract.

2   The Local Gravitational Field

Note to the lab technician: The computers to use are any of the following: Lassie, Dug, Buck, Ribsy, and Mudge.

Apparatus

(two stations):
vertical plank with electromagnets
steel balls (2/station)
Linux computers with Audacity installed
spirit levels

Goal

Make a high-precision measurement of the strength of the Earth's gravitational field, g, in Fullerton.

Introduction

When objects fall, and all forces other than gravity are negligible, we observe that the acceleration is the same, regardless of the object's mass, shape, density, or other properties. However, the acceleration does depend a little bit where on the earth we do the experiment, and even bigger variations in acceleration can be observed by, e.g., going to the moon. Thus, this acceleration can be considered as a property of space itself, and we can refer to it as the gravitational field in that region of space. Just as you would use a magnetic compass to find out about the magnetic field in the classroom, you can use dropping masses to find out about the gravitational field.

In this experiment, you'll measure the gravitational field, g, in the classroom to sufficiently high precision that, if everybody does a good job and we pool and average everyone's data to reduce random errors, we should be able to get a value that is measurably different from the generic world-average value you would find in a textbook.

Measuring g precisely

You will measure g, the acceleration of an object in free fall, using electronic timing techniques. The idea of the method is that you'll have two steel balls hanging underneath electromagnets at different heights.

You'll simultaneously turn off the two magnets by breaking the same electric circuit, causing the balls to drop at the same moment. The ball dropped from the lower height (h_1) takes a smaller time (Δt_1) to reach the floor, and the ball released from the greater height (h_2) takes a longer time (Δt_2). The time intervals involved are short enough that due to the limitations of your reflexes it is impossible to make good enough measurements with stopwatch. Instead, you will record the sounds of the two balls' impacts on the floor using the computer. The computer shows a graph in which the x axis is time and the y axis shows the vibration of the sound wave hitting the microphone. You can measure the time between the two visible "blips" on the screen. You will measure three things: h_1, h_2, and the time interval $\Delta t_2 - \Delta t_1$ between the impact of the second ball and the first. From these data, with a little algebra, you can find g.

The experiment would have been easier to analyze if we could simply drop a single ball and measure the time from when it was released to when it hit the floor. But since our timing technique is based on sound, and no sound is produced when the balls are released, we need to have two balls. If h_1, the height of the lower ball, could be made very small, then it would hit the floor at essentially the same moment the two balls were released (Δt_1 would equal 0), and $\Delta t_2 - \Delta t_1$ would be essentially the same as Δt_2. But we can't make h_1 too small or the sound would not be loud enough to detect on the computer.

When measuring h_1 and h_2, use the spirit level to get the two-meter stick accurately vertical.

Using the computer software

Start up the sound recording program, called Audacity. Set the record level on high, using the control marked $- \ldots +$ next to the microphone icon. Try recording a sound by clicking on the red "record" button.

To measure the time interval between the balls' impacts, you can measure the time at which each clap occurs, and then subtract. To measure a time, use the "I-beam" cursor and click on the feature you're interested in. Then click on the magnifying glass icon with the plus sign in it to zoom in. If you zoom in several times like this, you'll start to see that your
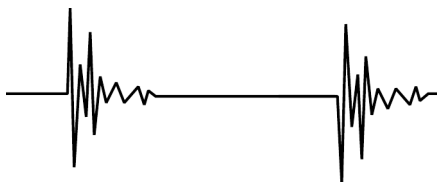
Two thumps, as recorded on the computer through the microphone.

positioning of the cursor was a little off. Correct this by clicking closer to the right place, and then zoom in some more. Continue this process of zooming in and correcting until you have measured the time as well as possible. Read the time from the scale at the top of the window.

Troubleshooting:

> You may get this error message: "Error while opening sound device. Please check the input device settings and the project sample rate." Quit the program and restart it.

> Some of the computers have very low or very high gain on their microphones. To work around this, do the following after you've recorded a sound: type control-A to select the whole recording, and then do Effect>Amplify; the default is to amplify the sound by the maximum amount, which is what you want. If the gain is too high, it may be necessary to lower it using the operating system's gain control, but every OS update seems to change the location of this control.

> If sound input isn't working, it may be because the wrong sound input device is selected. Right-click on the volume icon in the menu bar. Under Edit:Preferences, check Mic select, and under Options do Mic Select:Mic 0.

You should make a series of measurements, and make sure they agree at the level of a few times 10^{-4} s; if they don't, there's something wrong with your technique. Also, you should check that your result for g makes sense.

Here are some common problems that cause inconsistent or wrong results:

> The balls are brushing against the electrical wires as they fall.

> You're misidentifying the thumps.

> The surface the balls are dropping onto has dents in it.

You're not positioning the balls on the same spot on the magnets every time.

To drop the balls, you should break the circuit by pulling one of the banana-plug connectors out of the plug on the front panel of the power supply. Don't use the switch; if you use the switch, it takes some time for the magnetic field to decay, and the balls stick for a while before dropping.

Audacity will let you keep on making new recordings, stacking the graphs vertically. However, if you do this you will introduce significant timing errors. The reason is probably that Audacity is designed for use in multitrack recording of music, so it tries to play back the previously recorded tracks while recording the new one, and on cheap sound hardware this causes little timing glitches.

Analysis

Extract a value of g from your data.

Derive error bars on your result, using the techniques in appendices 2 and 3.

Self-Check

Extract the value of g, with error bars. Read Appendix 3 for information on how to do error analysis with propagation of errors; get help from your instructor if necessary.

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

P1 *If your instructor has assigned homework problem 27 from ch. 3 of Light and Matter,don't bother turning in another copy of your work for this prelab question.* Derive an equation for g in terms of the quantities you'll measure, which are h_1, h_2, and the time interval $\Delta t_2 - \Delta t_1$. The point of the lab is to measure g, so don't just say "well of course g is 9.8 m/s^2." (You should check your equation by using the answer checker for the homework problem.)
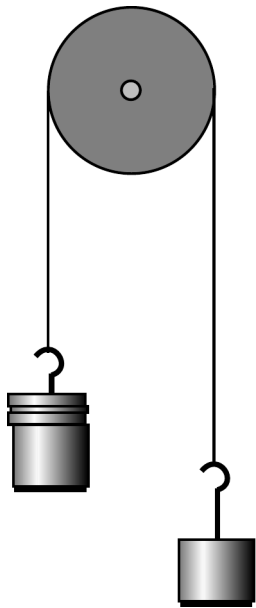
3   Newton's Second Law

Apparatus

pulley
spirit level
string
weight holders, not tied to string
two-meter stick
brass slotted weights (in lab benches in 415)
extra 1 g slotted weights
digital balance (Ohaus SP4001) 3
stopwatch
foam rubber cushions
graphite lubricant

Goal

Find the acceleration of unequal weights hanging
from a pulley.

Observations

Set up unequal masses on the two sides of the pulley,
and determine the resulting acceleration by measur-
ing how long it takes for the masses to move a certain
distance. Use the spirit level to make the plane of
the pulley vertical; otherwise you get extra friction.
Use relatively large masses (typically half a kg or a
kg each side) so that friction is not such a big force

in comparison to the other forces, and the inertia
of the pulley is negligible compared to the inertia of
the hanging masses. Do several different combina-
tions of masses, but *keep the total amount of mass
constant* and just divide it differently between the
two holders. Remember to take the masses of the
holders themselves into account. Make sure to per-
form your measurements with the longest possible
distance of travel, because you cannot use a stop-
watch to get an accurate measurement of very short
time intervals. The best results are obtained with
combinations of weights that give times of about 4
to 20 seconds. (Times greater than that range in-
dicate very low accelerations, which don't work well
because friction becomes a big effect.)

The brass weights were manufactured by the friendly
crack-smoking hillbillies at Glakad Science in North
Carolina. They have a tolerance (not disclosed in
the distributor's catalog) of $\pm 2\%$, meaning that a
500 g weight could be off by as much as ten grams!
Because of this, you will need to weigh your stacks
of weights on a digital balance to find out what they
really are.

Count your weights before you start and make sure
you have the full set listed on the box. During the
lab, keep the small ones on the holder. At the end
of lab, count your weights again.

Your stopwatch timing errors are determined by your
reflexes, which are presumably the same for all mass
combinations. You'll need to take a large number
of trials at some mass combination in order to find
this error accurately. It is pointless, however, to do
multiple trials for every mass combination.

Self-Check

Compare theoretical and experimental values of ac-
celeration for one of your mass combinations. Check
whether they come out fairly consistent.

Analysis

Use your measured times and distances to find the
actual acceleration, and make a graph of this versus
$M - m$. See appendix 4 re graphing. Show these ex-
perimentally determined accelerations as dots. Over-
laid on the same graph, show the theoretical equa-

tion as a line or curve, as in the examples in 4. Use propagation of errors (appendix 3) to determine error bars for your accelerations, and show them on your graph. Even though all the times have the same error bars, the accelerations will not. As in the examples in appendix 4, compare theory and experiment: did they agree to within your error bars?

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

P1 Criticize the following reasoning: The weight fell 1.0 m in 1 s, so $v = 1$ m/s, and $a = v/t = 1$ m/s^2.

P2 Since that won't work, plan how you really will determine your experimental accelerations based on your measured distance and times.

P3 *Skip this question if the corresponding homework problem has already been assigned.* Use Newton's second law to find a theoretical prediction for the acceleration a in terms of g and the masses M and m.

4   Vector Addition of Forces

Apparatus

force sensors 4/group
LabPro interface (in lab benches in 415)
string
butcher paper
500 g hooked weight (in lab benches in 415)
right-angle clamp
30-cm steel rod
spirit level

Goal

Test whether the vector sum of the forces acting on an object at rest is equal to zero.

Introduction

Modern physics claims that when a bridge, an earthquake fault, or an oak tree doesn't move, it is because the forces acting on it, which combine according to vector addition, add up to zero. Although this may seem like a reasonable statement, it was far from obvious to premodern scientists. Aristotle, for instance, said that it was the nature of each of the four elements, earth, fire, water and air, to return to its natural location. Rain would fall from the sky because it was trying to return to its natural location in the lakes and oceans, and once it got to its natural location it would stop moving because that was its nature.

When a modern scientist considers a book resting on a table, she says that it holds still because the force of gravity pulling the book down is exactly canceled by the normal force of the table pushing up on the book. Aristotle would have denied that this was possible, because he believed that at any one moment an object could have only one of two mutually exclusive types of motion: natural motion (the tendency of the book to fall to the ground, and resume its natural place), and forced motion (the ability of another object, such as the table, to move the book). According to his theory, there could be nothing like the addition of forces, because the object being acted on was only capable of "following orders" from one source at a time. The incorrect Aristotelian point of view has great intuitive appeal, and beginning physics students tend to make Aristotelian statements such as, "The table's force overcomes the force of gravity," as if the forces were having a contest, in which the victor annihilated the loser.
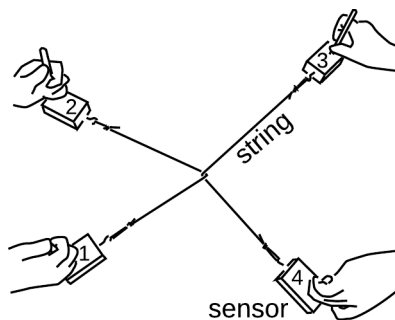
Observations

The apparatus consists of four taut strings, each pulling in a different direction in the horizontal plane. Each string is connected to a force sensor that can be read out on a computer, giving the magnitude of the force vector.

Set the force sensors on their 10 N scales using the switch on each sensor. Plug the sensors into channels 1 through 4 on the LabPro interface, and connect the interface to the USB port of the Windows machine. From the Start menu, go to All Programs, Vernier Software, Logger Pro 3.8. When you start up the software, it will automatically detect the four sensors and start displaying their readings.

Possible computer hassles:

> If the readings are displayed in a small font, you can make them bigger by clicking on the surrounding box and using the little black tabs to resize it.

> Normally the computer automatically detects the sensor and sets itself up properly. If this doesn't happen at all, you need to do the setup manually under Experiment:Set up sensors:Show all interfaces.

> If the computer incorrectly identifies all of the sensors as some other type of sensor, the problem is probably that one of the sensors has a damaged connected. Disconnect all the sensors and test them one at a time to find the damaged one.

If less than four sensors show up on the screen, the problem may be the same as the one described in the preceding item. A damaged connector on one sensor can cause other sensors not to work. Test the sensors one at a time.

Push and pull on the sensors, and verify that they can measure forces. What do you notice about the signs of the forces? You will notice that when you don't apply any force to a sensor, its reading is small, but not exactly zero. With the sensors placed horizontally, use the software's blue $\emptyset$ button to zero them all, so that this systematic error is eliminated. If you now rotate a sensor so that its hook is pointing down, you should see that the force reading changes by a certain amount, which is the weight of the hook, about 0.21 N.

Try pushing gently sideways on the force sensor. You will see that it doesn't sense the force. Each sensor can only measure the component of the force along its own axis.

After taking real data, you'll do a complete calibration of the sensors. They drift over time, so a calibration done right now wouldn't be useful. However, sometimes we have sensors that are off by a very large amount such as 50%, and in that case it's better to find out about that now, because it will make it look like your data don't even make sense. As a quick check, try pairs of sensors hooked together and pulling against each other, and make sure that they seem to at least approximately obey Newton's third law.

Cut off a piece of butcher paper. Tie strings to the sensors in the arrangement shown in the diagram. After some trial and error, I've found the following technique to minimize the pain of tying the strings. Let the sensors be 1, 2, 3, and 4, in order around the circle, as in the figure. The idea is to connect 1 and 2 with a string, and also 3 and 4, with the 1-2 string and 3-4 string wrapping around each other at an intersection-point at the center.[1] At each of the hooks, tie an overhand knot on a bight; this knot is easy to tie and throw over a hook, and will not slip.

To keep the sensors in place and apply tension to the strings, stick a pencil or a skinny pen into the hole in each one, and grip the pencil. This allows the sensor to naturally swing into position so that its axis is parallel to the string. Holding the sensors' housings

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[1]Because of friction at the intersection, the four tensions can all be different. If the strings were frictionless, we would be guaranteed to have $T_1 = T_2$ and $T_3 = T_4$ with this arrangement, and that probably wouldn't allow us to get the right forces for equilibrium.

in your hands doesn't work, because it results in a misalignment.

Avoid a symmetric arrangement of the strings (e.g., don't space them all 90 degrees apart), and don't make any forces collinear with each other.

overhand on a bight

The angles are the main source of random error. Once you have a setup that's adjusted approximately the way you want, you can draw lines on the paper with a ruler, then carefully locate the strings above the lines. The angles of the lines on the paper are easy to measure accurately.

Try to get the forces to be roughly 3 to 7 N. Although the sensors are labeled "10 N," some of them actually max out at 9 or 9.5 N. The sensors will not give any error or warning if this limit is exceeded. They will simply max out at some reading, and this reading will be incorrect. When increasing the tension in a string, keep an eye on the reading on the screen; if it stops increasing, you've exceeded that sensor's limit. Once you think you have a good equilibrium, check that when you increase the tension a little, the reading actually goes up.

Before taking your final data, temporarily relax the tension on the strings and redo the zeroing of the sensors, which we have found will drift over time.

Take all the data you will need in order to carry out the vector addition of the four forces. To decrease the random errors in the force measurements, you can tell the software to average the data over a period of time. Click on the Collect button to make a graph. Then highlight part of the graph, and go to Analyze:Statistics.

As a cross-check in case confusion arises later about how your angles were defined, which force was which, etc., write down on the butcher paper which sensor was which. Also, mark the sensors 1, 2, 3, and 4 with masking tape.

The sensors differ from each other quite a bit in their calibration, with four freshly zeroed sensors giving readings that covered a range of about 5% when used to measure a 500 g weight. To eliminate this error, set up the 30-cm steel bar horizontally and mount all four sensors on it vertically, with their hooks point-

ing down. Zero the sensors again so that the weights of the hooks are tared out. Hang the 500 g weight from each sensor, and use these readings to rescale each sensor's reading from the vector addition setup by the proper amount.

Self-Check

Do a preliminary rough analysis of your results using graphical addition.

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

P1 Describe a typical scale that you might use for drawing force vectors on a piece of paper, e.g., how long might you choose to make a 1-N force? Assume that your forces are from 5 to 10 newtons, and pick a scale that results in a drawing that is as big as it can be (for maximum precision) while still fitting on a piece of paper.

P2 Thinking ahead to your analytic addition of the forces, which of the following would be the most convenient form in which to have your angle measurements? — (1) the angles of the four "pie slices," (2) the angle of each string measured clockwise around the table, or (3) the angle of each string measured counterclockwise around the table.

Analysis

Calculate the magnitude of vector sum of the forces on the ring both graphically and analytically, using the corrected data. Use the graphical result to check that the analytic calculations were correct.

Propagation of error for this lab could in principle be very time-consuming, since you would have to redo the vector addition eight times for the eight pieces of raw data (four angles and four magnitudes). To avoid this, take the following shortcuts: (1) Whichever is larger, $F_{total,x}$ or $F_{total,y}$, just do error analysis on that component. (2) The random errors in the result are dominated by the errors in the angles, so don't bother propagating the error from the magnitudes.

Are your results consistent with theory, taking into account the random errors involved? Perform a sta-

tistical test like the one in the example on p. 51 ("How significant?").

5   Acceleration In Two Dimensions

Apparatus

air track (small)
cart
LabPro-compatible photogates (in lab benches in 415)
computer
air blowers (in cupboards under lab benches)
vernier calipers (for measuring wood blocks)
wood blocks 2-3/group
angle brackets

Goal

Test whether the acceleration of gravity acts like a vector.

Introduction

Vectors rule the universe. Entomologists say that God must have had an inordinate fondness for beetles, because there are so many species of them. Well, God must also have had a special place in her heart for vectors, because practically every natural phenomenon she invented is a vector: gravitational acceleration, electric fields, nuclear forces, magnetic fields, all the things that tie our universe together are vectors.
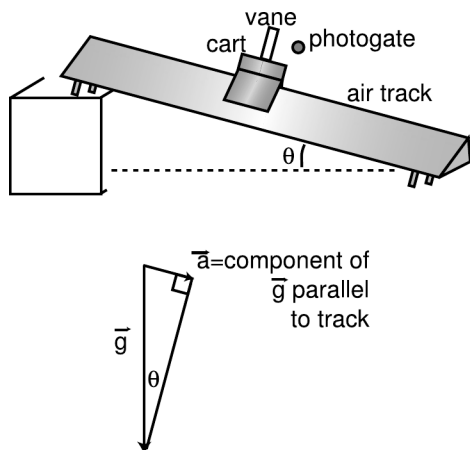
Setup

The idea of the lab is that if acceleration really acts like a vector, then the cart's acceleration should equal the component of the earth's gravitational acceleration vector that is parallel to the track, because the cart is only free to accelerate in the direction along the track. There is almost no friction, since the cart rides on a cushion of air coming through holes in the track.

The speed of the cart at any given point can be measured as follows. The photogate consists of a light and a sensor on opposite sides of the track. When the cart passes by, the cylindrical vane on top blocks the light momentarily, keeping light from getting to the sensor. The computer detects the electrical signal from the sensor, and records the amount of time, t_b, for which the photogate was blocked. Given t_b, you can determine the approximate speed that cart had when it passed through the photogate. The computer software for measuring the time is on the Linux computers in 415. Plug the photogate into the DIG1 plug on the interface box, and connect the interface box to the computer using the USB cable. Double-click on the icon that says "photogate."

Observations

The basic idea is to release the cart at a distance x away from the photogate. The cart accelerates, and you can determine its approximate speed, v, when it passes through the photogate. (See prelab question P1.)

From v and x, you can find the acceleration. You will take data with the track tilted at several different angles, to see whether the cart's acceleration always equals the component of g parallel to the track.

You can level the track to start with by adjusting the screws until the cart will sit on the track without accelerating in either direction.

The distance x can be measured from the starting position of the cart to half-way between the point where it first blocks the photogate and the point where it unblocks the photogate. You can determine where these positions are by sliding the cart into the photogate and watching the red LED on the top of the photogate, which lights up when it is blocked.

Hints:

    Keep in mind that if the cart rebounds at the bottom of the track and comes back up through

the gate, you will get a second, bogus time reading.

Note that you have no way to measure accurately to the total amount of time over which the cart picked up speed (which would be several seconds) — what you measure is the very short time required for the cart to pass through the photogate.

Release the cart by hand after starting up the air pump. If you leave the cart on the track and then turn on the pump, there will be a period of time when the pump is first starting up, and the cart will drag.

The variable x actually changes a little when you change θ, so don't just assume it's always the same.

Once you have data at your first angle, check whether theory and experiment agree reasonably well.

You'll use the photogates again in lab 10, so make sure you understand the technique thoroughly, and take notes on it so you'll remember how it's done.

We use wood blocks to raise the track to various angles. To measure the thickness of the blocks, you will use special calipers that have a vernier scale, which is a tricky device that allows you to measure distances to a precision of about a tenth of a millimeter — about ten times better than eyeball precision with a ruler. Your instructor will teach you how to use the vernier scale.

The accurate measurement of the width of the vane, w, is critical to the whole experiment. This has already been done for you, using the technique described at the end of the lab. The results, in cm, are as follows.

1A	2.13	1B	2.18	1C	2.05
2A	2.11	2B	2.13	2C	2.14
3A	2.10	3B	2.18	3C	2.07
4A	2.10	4B	2.12	4C	2.14
5A	2.15	5B	2.18	5C	2.14
6A	2.11	6B	2.15	6C	2.19
7A	2.15			7C	2.16

Technique for measuring the width of the vane

Students don't need to do this.

Ideally we would like to simply close the vernier calipers snugly on the cylindrical vane and take a

reading of its diameter. This is the way the calipers are intended to be used. However, the beam of the photogate has a finite width, and the electronics in the photogate are only roughly calibrated to turn on and off when the edge of the vane intersects the center of the beam, blocking half its light. Therefore, I had one of my classes in 2011 use an arrangement like the one below to determine the effective width of the vane when it is used with your own photogate. The idea is to use the moving jaw of the calipers to push the cart from the point where it enters your photogate to the point where it exits, as determined by the LED on top. I have found that the photogates differ from one another by about a millimeter, so that if you don't do this calibration with your own photogate, your velocities will be off by as much as 5%.
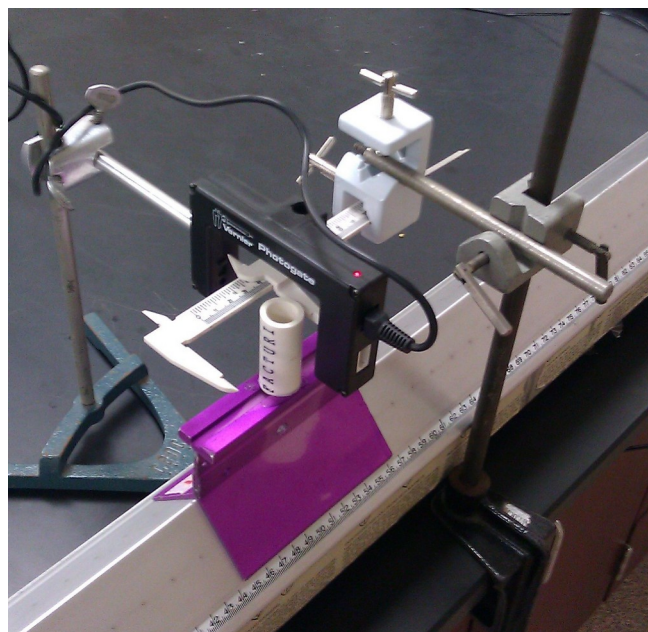
Self-Check

Find the theoretical and experimental accelerations for one of your angles, and see if they are roughly consistent.

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

P1   *Skip this question if the corresponding home-*
work problem has already been assigned. (a) If w is
the width of the vane, and t_b is defined as suggested
above, what is the speed of the cart when it passes
through the photogate? (b) Based on v and x, how
can you find a?

P2   It is not possible to measure θ accurately with
a protractor. How can θ be determined based on
the distance between the feet of the air track and
the height of the wood block? To figure out how to
actually apply the trig to the lab, you will need to
draw a side view of the track with enough detail to
show the track as a rectangle and the feet sticking
out.

Analysis

Extract the acceleration for each angle at which you
took data. Make a graph with θ on the x axis and
acceleration on the y axis. Show your measured ac-
celerations as points, and the theoretically expected
dependence of a on θ as a smooth curve.

Error analysis is not required for this lab, because
the random errors are small compared to systematic
errors such as the imperfect leveling of the track,
friction, warping of the track, and the measurement
of w. Appendix 4 shows some examples of how to
compare theory and experiment on a graph.

6 Conservation Laws

Apparatus

Part A: vacuum pump (Lapine) 1
electronic balance (Ohaus Scout Pro) 1
plastic-coated flask 1/group
Part B: beaker 1/group
propyl alcohol 200 mL/group
canola oil 200 mL/group
funnels 2/group
100-mL volumetric flask 1/group
1-ml pipette and bulb 1/group
magnetic stirrer 1/group
triple-beam balance 1/group

Goal

People believe that objects cannot be made to disappear or appear. If you start with a certain amount of matter, there is no way to increase or decrease that amount. This type of rule is called a conservation law in physics, and this specific law states that the amount of matter is conserved, i.e., must stay the same. In order to make this law scientifically useful, we must define more carefully how the "amount" of a substance is to be defined and measured numerically. Specifically, there are two issues that scientifically untrained people would probably not agree on:

> Should air count as matter? If it has weight, then it probably should count. In this lab, you will find out if air has weight, and, if so, measure its density.

> Should the amount of a substance be defined in terms of volume, or is mass more appropriate? In this lab, you will determine whether mass and/or volume is conserved when water and alcohol are mixed.

Introduction

Styles in physics come and go, and once-hallowed principles get modified as more accurate data come along, but some of the most durable features of the science are its conservation laws. A conservation law is a statement that something always remains constant when you add it all up. Most people have a general intuitive idea that the amount of a substance is conserved. That objects do not simply appear or disappear is a conceptual achievement of babies around the age of 9-12 months. Beginning at this age, they will try to retrieve a toy that they have seen being placed under a blanket, rather than just assuming that it no longer exists. Conservation laws in physics have the following general features:

> Physicists trying to find new conservation laws will try to find a measurable, numerical quantity, so that they can check quantitatively whether it is conserved. One needs an operational definition of the quantity, meaning a definition that spells out the operations required to measure it.

> Conservation laws are only true for closed systems. For instance, the amount of water in a bottle will remain constant as long as no water is poured in or out. But if water can get in or out, we say that the bottle is not a closed system, and conservation of matter cannot be applied to it.

> The quantity should be additive. For instance, the amount of energy contained in two gallons of gasoline is twice as much as the amount of energy contained in one gallon; energy is additive. An example of a non-additive quantity is temperature. Two cups of coffee do not have twice as high a temperature as one cup.

> Conservation laws always refer to the total amount of the quantity when you add it all up. If you add it all up at one point in time, and then come back at a later point in time and add it all up, it will be the same.

How can we pin down more accurately the concept of the "amount of a substance"? Should a gallon of shaving cream be considered "more substantial" than a brick? At least two possible quantities come to mind: mass and volume. Is either conserved? Both? Neither? To find out, we will have to make measurements.

We can measure mass by the "see-saw method" — when two children are sitting on the opposite sides of a see-saw, the less massive one has to move farther out from the fulcrum to make it balance. If we

enslave some particular child as our permanent mass standard, then any other child's mass can be measured by balancing her on the other side and measuring her distance from the fulcrum. A more practical version of the same basic principle that does not involve human rights violations is the familiar pan balance with sliding weights.

Volume is not necessarily so easy to measure. For instance, shaving cream is mostly air, so should we find a way to measure just the volume of the bubbly film itself? Precise measurements of volume can most easily be done with liquids and gases, which conform to a vessel in which they are placed.

Should a gas, such as air, be counted as having any substance at all? Empedocles of Acragas (born ca. 492 BC) was the originator of the doctrine that all material substances are composed of mixtures of four elements: earth, fire, water and air. The idea seems amusingly naive now that we know about the chemical elements and the periodic table, but it was accepted in Europe for two thousand years, and the inclusion of air as a material substance was actually a nontrivial concept. Air, after all, was invisible, seemed weightless, and had no definite shape. Empedocles decided air was a form of matter based on experimental evidence: air could be trapped under water in an inverted cup, and bubbles would be released if the cup was tilted. In China around 300 BC, Zou Yan came up with a similar theory, and his five elements did not include air.

Does air have weight? Most people would probably say no, since they do not feel any physical sensation of the atmosphere pushing down on them. A delicate house of cards remains standing, and is not crushed to the floor by the weight of the atmosphere.

But compare that to the experience of a dolphin. A dolphin might contemplate a tasty herring suspended in front of it and conjecture that water had no weight, because the herring did not involuntarily shoot down to the sea floor because of the weight of the water overhead. Water does have weight, however, which a sufficiently skeptical dolphin physicist might be able to prove with a simple experiment. One could weigh a 1-liter metal box full of water and then replace the water with air and weigh it again. The difference in weight would be the difference in weight between 1 liter of water and 1 liter of air. Since air is much less dense than water, this would approximately equal the weight of 1 liter of water.

Our situation is similar to the dolphin's, as was first appreciated by Torricelli, whose experiments led him to conclude that "we live immersed at the bottom of a sea of...air." A human physicist, living her life immersed in air, could do a similar experiment to find out whether air has weight. She could weigh a container full of air, then pump all the air out and weigh it again. When all the matter in a container has been removed, including the air, we say that there is a vacuum in the container. In reality, a perfect vacuum is very difficult to create. A small fraction of the air is likely to remain in the container even after it has been pumped on with a vacuum pump. The amount of remaining air will depend on how good the pump is and on the rate at which air leaks back in to the container through holes or cracks.

Cautions

Please do not break the glassware! The vacuum flasks and volumetric flasks are expensive.

The alcohol you will be using in this lab is chemically different from the alcohol in alcoholic drinks. It is poisonous, and can cause blindness or death if you drink it. It is also flammable.

Observations

A Density of air

You can remove the air from the flask by attaching the vacuum pump to the vacuum flask with the rubber and glass tubing, then turning on the pump. You can use the scale to determine how much mass was lost when the air was evacuated.

Make any other observations you need in order to find out the density of air and to estimate error bars for your result.

B Is volume and/or mass conserved when two fluids are mixed?

The idea here is to find out whether volume and/or mass is conserved when water and alcohol are mixed. The obvious way to attempt this would be to measure the volume and mass of a sample of water, the volume and mass of a sample of alcohol, and their volume and mass when mixed. There are two problems with the obvious method: (1) when you pour one of the liquids into the other, droplets of liquid will be left inside the original vessel; and (2) the most accurate way to measure the volume of a liquid is with a volumetric flask, which only allows one specific, calibrated volume to be measured.

Here's a way to get around those problems. Put the

alcohol

oil
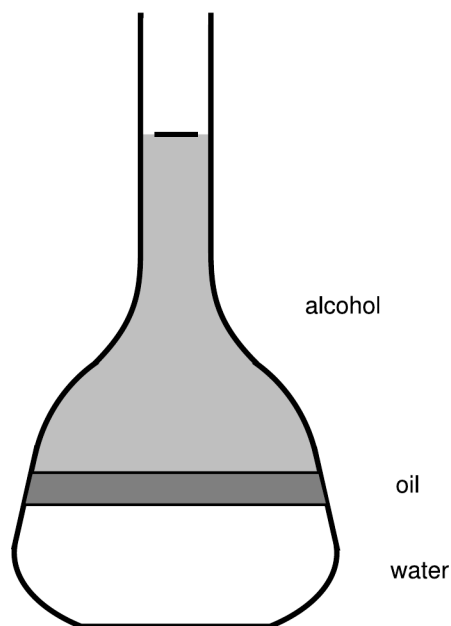
water

magnetic stirrer inside the flask. Pour water through a funnel into a volumetric flask, filling it less than half-way. (Do not use the pipette to transfer the water.) A common mistake is to fill the flask more than half-way. Now pour a thin layer of cooking oil on top. Cooking oil does not mix with water, so it forms a layer on top of the water. (Set aside one funnel that you will use only for the oil, since the oil tends to form a film on the sides.) Finally, gently pour the alcohol on top. Alcohol does not mix with cooking oil either, so it forms a third layer. By making the alcohol come exactly up to the mark on the calibrated flask, you can make the total volume very accurately equal to 100 mL. In practice, it is hard to avoid putting in too much alcohol through the funnel, so if necessary you can take some back out with the pipette.

If you put the whole thing on the balance now, you know both the volume (100 mL) and the mass of the whole thing when the alcohol and water have been kept separate. Now, mix everything up with the magnetic stirrer. The water and alcohol form a mixture. You can now test whether the volume or mass has changed.

If the mixture does not turn out to have a volume that looks like exactly 100 mL, you can use the following tricks to measure accurately the excess or deficit with respect to 100 mL. If it is less than 100 mL, weigh the flask, pipette in enough water to bring it up to 100 mL, weigh it again, and then figure out what mass and volume of water you added based on the change in mass. If it is more than 100 mL,

weigh the flask, pipette out enough of the mixture to bring the volume down to 100 mL, weigh it again, and make a similar calculation using the change in mass and the density of the mixture. If you need to pipette out some of the oily mixture, make sure to wash and rinse the pipette thoroughly afterwards.

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

P1 Suppose that the initial mass in part B is $m_i = 280.3 \pm 0.3$ g and the final mass (after mixing) is $m_f = 281.8 \pm 0.3$ g, with the error bars as determined in appendix 2. Calculate the change in mass Δm, and use the technique described in appendix 3 to find the error bars on this result.

P2 As in the example in appendix 2, find out by how many standard deviations this result for Δm differs from zero, and give a probabilistic interpretation of whether or not this is consistent with conservation of mass.

Self-Check

Do a quick analysis of both parts without error analysis. Plan how you will do your error analysis.

Analysis

A. If your results show that air has weight, determine the (nonzero) density of air, with an estimate of your random errors.

B. Decide whether volume and/or mass is conserved when alcohol and water are mixed, taking into account your estimates of random errors.
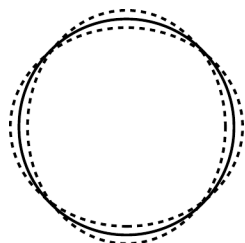
7 Conservation of Energy

Apparatus

air track
cart
springs (steel, 1.5 cm diameter)
photogate
computer
aluminum rods, ~ 45 cm
spirit levels
air blowers
spring scales
string
balances
scissors

Goal

Test conservation of energy for an object oscillating around an equilibrium position.


This could be a vibration of the sun, a water balloon, or a nucleus.

Introduction

One of the most impressive aspects of the physical world is the apparent permanence of so many of its parts. Objects such as the sun or rocks on earth have remained unchanged for billions of years, so it might seem that they are in perfect equilibrium, with zero net force on each part of the whole. In reality, the atoms in a rock do not sit perfectly still at an equilibrium point — they are constantly in vibration about their equilibrium positions. The unchanging oblate shape of the sun is also an illusion. The sun is continually vibrating like a bell or a jiggling water balloon, as shown in the (exaggerated) figure. The nuclei of atoms also jiggle spontaneously like little water balloons. The fact that these types of motion continue indefinitely without dying out or building up relates to conservation of energy, which forbids them to get bigger or smaller without transferring energy in or out.
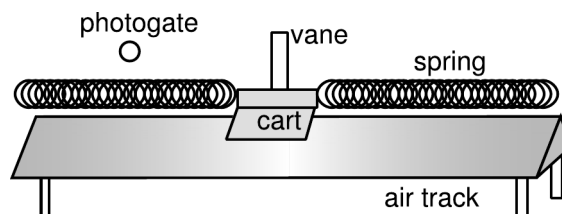
Our model of this type of oscillation about equilibrium will be the motion of a cart on an air track between two springs. The sum of the forces exerted by the two springs should at least approximately obey Hooke's law,

$$F = -kx \qquad ,$$

where the equilibrium point is at $x = 0$. The negative sign means that if the object is displaced in the positive direction, the force tends to bring it back in the negative direction, towards equilibrium, and vice versa. Of course, there are no actual springs involved in the sun or between a rock's atoms, but we can still learn about this type of situation in a lab experiment with a mass attached to a spring. In this lab, you will study how the changing velocity of the object, in this case a cart on an air track, can be understood using conservation of energy. Recall that for a constant force, the potential energy is simply $-Fx$, but for a force that is different at different locations, the potential energy is minus the area under the curve on a graph of F vs. x. In the present case, the area formed is a triangle with base $= x$, height $= kx$, and
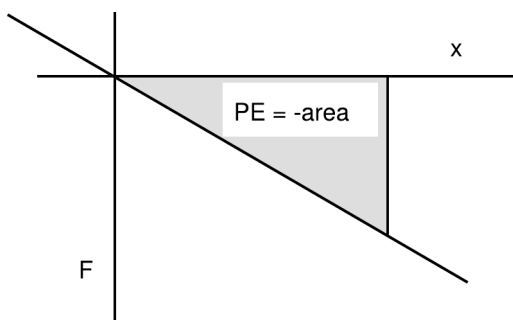
$$\text{area} = \frac{1}{2}\text{base} \cdot \text{height}$$
$$= -\frac{1}{2}kx^2$$

(counted as negative area because it lies below the x axis), so the potential energy is

$$PE = \frac{1}{2}kx^2 \qquad .$$

Conservation of energy, $PE + KE = $ constant, requires the constancy of

$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = E_{total} \qquad .$$

PE = -area

Preliminary Observations

You should do both of the following methods of determining the spring constant.

Determining the spring constant: method 1

Pull the cart to the side with a spring scale, and determine the spring constant, which is defined as minus the slope of the F versus x graph. Don't detach either spring; just measure the whole system to find the combined spring constant of the two springs, k. Since you'll only use method 1 as a rough check against the more precise method 2, it is a waste of time to take more than two data points, and you can use these points to find the slope without actually having to make a graph.

Determining the spring constant: method 2

The second technique for determining k is to raise one end up and observe how far the cart's equilibrium is displaced. This method is more accurate than method 1, but your k value from method 1 is still useful as a check.

You can determine the angle θ to which the track has been raised by the same trig you used in lab 5. At this angle, the component $mg\sin\theta$ of the gravitational force that is parallel to the track cancels out the force kx from the springs. Since you know m, g, θ, and x, you can determine k.

The main limitation on the precision of this measurement is that the displacement x is fairly small. To maximize your precision, raise the track to as high an angle as is practical. A good method is to prop it up with the aluminum rod, using the spirit level to make sure the rod is vertical.

Observations

The technique is essentially the same as in lab 5, which you will want to review.

Before you start taking actual data, check whether you have excessive friction by letting the computer record data while the cart vibrates back and forth a few times through the photogate. If the air track is working right, all the time measurements should be nearly the same, but if the data show the cart slowing down a lot from one vibration to the next, then you have a problem with friction. A drop in velocity of about 1% over a half-cycle is reasonable.

Measure the velocity of the cart for many different values of x by moving the photogate to various positions. Make sure you always release the cart from rest at the same point, and when you are initially choosing this release point, make sure that it is not so far from the center that the springs are completely bunched up or dragging on the track. Don't forget that the x you use in the potential energy should be the distance from the equilibrium position to the position where the vane is centered on the photogate — if you don't think about it carefully, it's easy to make a mistake in x equal to half the width of the vane. See lab 5.

Self-Check

Calculate the energies at the extremes, where $PE = 0$ and $KE = 0$, and see whether the energy is staying roughly constant. You should do this self-check as early as possible in the lab, so that you can make sure you're not spending lots of time collecting data that turn out to be bogus.

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

P1 Find the value of x from the figure below. (I've made the centimeter scale unrealistic for readability — the real track is more than a meter long, not 14 centimeters.)

P2 In order to do the self-check, where would you have to put the photogate?

P3 Plan how you will estimate the random errors
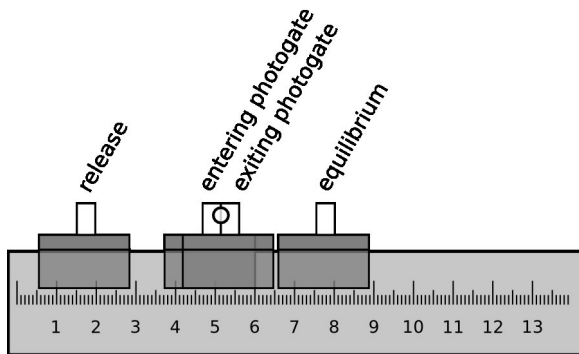
on your raw data. (Random errors here would refer to something that would cause independent errors that could be up or down for any individual data point — not things like the measurement of k, which you do once for the whole experiment.)

Analysis

Graph PE, KE, and the total energy as functions of x, with error bars (see appendices 2, 3, and 4), all overlaid on the same plot. Make sure to include the points with $KE = 0$ and $PE = 0$.

As a shortcut in your error analysis, it's okay if you do the error analysis for your most typical data-point, in which the energy is split roughly 50-50 between PE and KE, and then assume that the same error bars on PE, KE, and total energy apply to all the other points on the graph as well.

As sources of error, you have certain things, such as the spring constant k, that you determine once for the entire lab. These are more like systematic errors rather than random errors. The error bars you draw on the graph are meant to represent random errors, i.e., errors that could cause random scatter in the points. Therefore you should not include systematic errors in these error bars.

Discuss whether you think conservation of energy has been verified. See the examples in appendix 4 re how to statistically interpret this type of comparison of data with a curve on a graph.

8   Conservation of Momentum

Apparatus

computer with Logger Pro software
track
2 carts with magnets and mounting brackets for force sensors
1-kg weight
500 g slotted weight (in lab benches in 415)
masking tape
2 force sensors with rubber corks

The format of this lab is informal. You can write your answers in the blanks in the lab manual for parts A-G, and you don't need to write up anything at the end. I'll just discuss the physics verbally with your group as a whole.

Qualitative Observations

Level the track.

First you're going to observe some collisions between two carts and see how conservation of momentum plays out. If you really wanted to take numerical data, it would be a hassle, because momentum depends on mass and velocity, and there would be four different velocity numbers you'd have to measure: cart 1 before the collision, cart 1 after the collision, cart 2 before, and cart 2 after. To avoid all this complication, the first part of the lab will use only visual observations.

Try gently pressing the two carts together on the track. As they come close to each other, you'll feel them repelling each other! That's because they have magnets built into the ends. The magnets act like perfect springs. For instance, if you hold one cart firmly in place and let the other one roll at it, the incoming cart will bounce back at almost exactly the same speed. It's like a perfect superball.

A Equal masses, target at rest, elastic collision

Roll one cart toward the other. The target cart is initially at rest. Don't hold the target cart in place. Conservation of momentum reads like this,

$M \times$ _____ $+ M \times$ _____
$=?$ $M \times$ _____ $+ M \times$ _____   ,

where the two blanks on the left stand for the two carts' velocities before the collision, and the two

blanks on the right are for their velocities after the collision. All conservation laws work like this: the total amount of something remains the same. You don't have any real numbers, but just from eye-balling the collision, what seems to have happened? Let's just arbitrarily say that the mass of a cart is one unit, so that wherever it says "M x" in the equation, you're just multiplying by one. You also don't have any numerical values for the velocities, but suppose we say that the initial velocity of the incoming cart is one unit. Does it look like conservation of momentum was satisfied?

B Mirror symmetry

Now reenact the collision from part A, but do everything as a mirror image. The roles of the target cart and incoming cart are reversed, and the direction of motion is also reversed.

$M \times$ _____ $+ M \times$ _____
$=?$ $M \times$ _____ $+ M \times$ _____   ,

What happens now? Note that mathematically, we use positive and negative signs to indicate the direction of a velocity in one dimension.

C An explosion

Now start with the carts held together, with their magnets repelling. As soon as you release them, they'll break contact and fly apart due to the repulsion of the magnets.

$M \times$ _____ $+ M \times$ _____
$=?$ $M \times$ _____ $+ M \times$ _____   ,

Does momentum appear to have been conserved?

D Head-on collision

Now try a collision in which the two carts head towards each other at equal speeds (meaning that one cart's initial velocity is positive, while the other's is negative).

$M \times$ _____ $+ M \times$ _____
$=?$ $M \times$ _____ $+ M \times$ _____   ,

E Sticking

Arrange a collision in which the carts will stick together rather than rebounding. You can do this by letting the velcro ends hit each other instead of the magnet ends. Make a collision in which the target is initially stationary.

$M \times$ _____ $+ M \times$ _____
$=?$ $M \times$ _____ $+ M \times$ _____ ,

The collision is no longer perfectly springy. Did it seem to matter, or was conservation of momentum still valid?

F Hitting the end of the track

One end of the track has magnets in it. Take one cart off the track entirely, and let the other cart roll all the way to the end of the track, where it will experience a repulsion from the fixed magnets built into the track. Was momentum conserved? Discuss this with your instructor.

G Unequal masses

Now put a one kilogram mass on one of the carts, but leave the other cart the way it was. Attach the mass to it securely using masking tape. A bare cart has a mass of half a kilogram, so you've now tripled the mass of one cart. In terms of our silly (but convenient) mass units, we now have masses of one unit and three units for the two carts. Make the triple-mass cart hit the initially stationary one-mass-unit cart.

$3M \times$ _____ $+ M \times$ _____
$=?$ $3M \times$ _____ $+ M \times$ _____ ,

These velocities are harder to estimate by eye, but if you estimate numbers roughly, does it seem possible that momentum was conserved?

Quantitative Observations

Now we're going to explore the reasons why momentum always seems to be conserved. Parts H and I will be demonstrated by the instructor for the whole class at once.

Attach the force sensors to the carts, and put on the rubber stoppers. Make sure that the rubber stoppers are positioned sufficiently far out from the body of the cart so that they will not rub against the edge of the cart. Put the switch on the sensor in the +10 N position. Plug the sensors into the CH1 and CH2 ports on the interface box, and plug the interface box into the Windows computer. Start up the Logger Pro software (version 3), and do File>Open>Probes and Sensors>Force Sensors>Dual Range Forrce>2-10 N Dual Range. Go to Experiment>Data Collection, and set Sampling rate to 250 samples/s.

Tell the computer to zero the sensors. Try collecting data and pushing and pulling on the rubber stopper. You should get a graph showing how the force went up and down over time. The sensor uses negative numbers (bottom half of the graph) for forces that squish the sensor, and positive numbers (top half) for forces that stretch it. Try both sensors, and make sure you understand what the red and blue traces on the graph are showing you.

H. Put the extra 1-kilogram weight on one of the carts. Put it on the track by itself, without the other cart. Try accelerating it from rest with a gentle, steady force from your finger. You'll want to set the collection time to a longer period than the default. Position the track so that you can walk all the way along its length (not diagonally across the bench).

What does the graph on the computer look like?

I. Now repeat H, but use a more rapid acceleration to bring the cart up to the same momentum. Sketch a comparison of the graphs from parts H and I.

Discuss with your instructor how this relates to momentum.

J. You are now going to reenact collision A, but don't do it yet! You'll let the carts' rubber corks bump into each other, and record the forces on the sensors. The carts will have equal mass, and both forces will be recorded simultaneously. Before you do it, predict what you think the graphs will look like, and show your sketch to your instructor.

Switch both sensors to the +50 N position, and open the corresponding file on the computer.

Zero the sensors, then check the calibration by balancing a 500 g slotted weight on top, taking data, zooming in, and putting the mouse cursor on the graph. You will probably find that the absolute calibration of the sensor is very poor when it's used on the 50 N scale; keep this in mind when interpreting your results from the collision.

Now try it. To zoom in on the relevant part of the graph, use the mouse to draw a box, and then click on the magnifying glass icon. You will notice by eye that the motion after the collision is a tiny bit different than it was with the magnets, but it's still pretty similar. Looking at the graphs, how do you explain the fact that one cart lost exactly as much momentum as the other one gained? Discuss this with your instructor before going on.

K. Now imagine – but don't do it yet – that you are going to reenact part G, where you used unequal masses. Sketch your prediction for the two graphs, and show your sketch to your instructor before you go on.

Now try it, and discuss the results with your instructor.

9   Torque

Apparatus

meter stick with holes drilled in it 1/group
spring scales, calibrated in newtons
slotted weights (in lab benches in 415)
holder for slotted weights
string
protractors
Ohaus balances

Goal

Test whether the total force and torque on an object at rest both equal zero.

Introduction

It is not enough for a boat not to sink. It also must not capsize. This is an example of a general fact about physics, which is also well known to people who overindulge in alcohol: if an object is to be in a stable equilibrium at rest, it must not only have zero net force on it, to keep from picking up momentum, but also zero net torque, to keep from acquiring angular momentum.
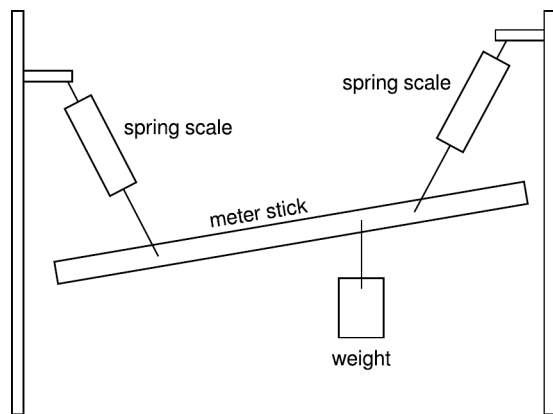
Observations

Weigh your meter stick before you do anything else; they don't all weigh the same amount.

Construct a setup like the one shown above. Avoid any symmetry in your arrangement. There are four forces acting on the meter stick:

   $\mathbf{F}_H$ = the weight hanging underneath
   $\mathbf{F}_M$ = Earth's gravity on the meter stick itself
   $\mathbf{F}_L$ = tension in the string on the left
   $\mathbf{F}_R$ = tension in the string on the right

Each of these forces also produces a torque.

In order to determine whether the total force is zero, you will need enough raw data so that for each torque you can extract (1) the magnitude of the force vector, and (2) the direction of the force vector. In order to add up all the torques, you will have to choose an axis of rotation, and collect enough raw data to be able to determine for each force (3) the distance from the axis to the point at which the force is applied to the ruler, and (4) the angle between the force vector and the line connecting the axis with the point where the force is applied. Note that the meter stick's own weight can be thought of as being applied at its center of mass.

The meter stick has holes drilled in it that you can use to attach the strings. You can make your analysis simpler by tying the knots as shown below, so that all the forces act at points along the center-line of the stick.
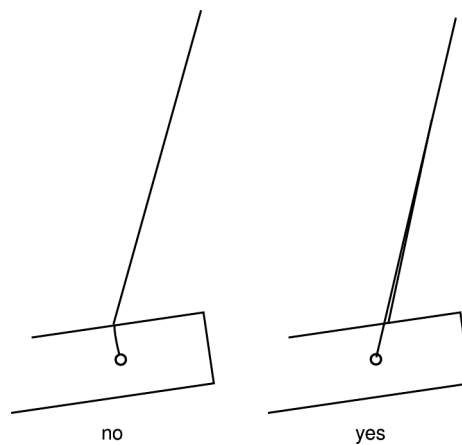
You have a selection of spring scales, so use the right one for the job — don't use a 20 N scale to measure 0.8 newtons, because it will not be possible to read it accurately. Optimize your precision by choosing conditions that come as close as possible to maxing out the scales. For each spring scale, hang a known weight from it, and adjust the calibration tab so that

the scale gives the correct result. If you need to swap in a new spring scale, don't forget to calibrate it.

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

P1   You have complete freedom in defining what point is to be considered the axis of rotation — if one choice of axis causes the total torque to be zero, then any other choice of axis will also cause the total torque to be zero. It is possible to simplify the analysis by choosing the axis so that one of the four torques is zero. Plan how you will do this.

P2   All the torques will be tending to cause rotation in the same plane. You can therefore use plus and minus signs to represent clockwise and counterclockwise torques. Choose which one you'll call positive. Using your choice of axis, which of the four torques, τ_H, τ_M, τ_L, and τ_R, will be negative, which will be positive, and which will be zero?

Analysis

Determine the total force and total torque on the meter stick. For the forces, I think a graphical calculation will be easier than an analytic one.

Finally, repeat your calculation of the total torque using a different point as your axis. Although you're normally expected to do your analysis completely independently, for this lab it's okay if you find the total torque for one choice of axis, and your lab partners do the calculation for their own choices. Present both results in your own abstract.

Error analysis is not required. For extra credit, you can do error analysis for your total torque.

10   **Resonance**

Apparatus

vibrator 1/group
stopwatch 1/group
multimeter (in lab bench) 1/group
banana plug cables
Leybold 521 545 17-volt DC power supplies (or BK
Precision 1505)
24 V AC power supplies
spray lubricant
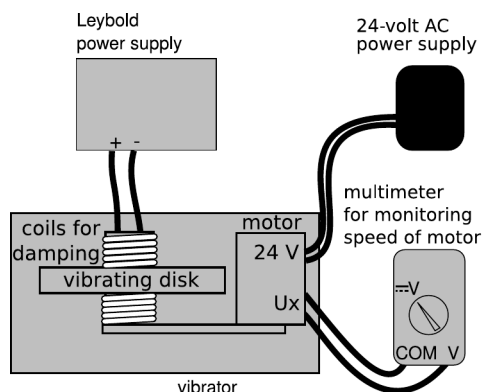
Electrical setup, top view.

Goals

Observe the phenomenon of resonance.

Investigate how the width of a resonance depends on the amount of damping.

Introduction

To break a wine glass, an opera singer has to sing
the right note. To hear a radio signal, you have to
be tuned to the right frequency. These are examples
of the phenomenon of resonance: a vibrating system
will respond most strongly to a force that varies with
a particular frequency.

variable-speed
electric motor
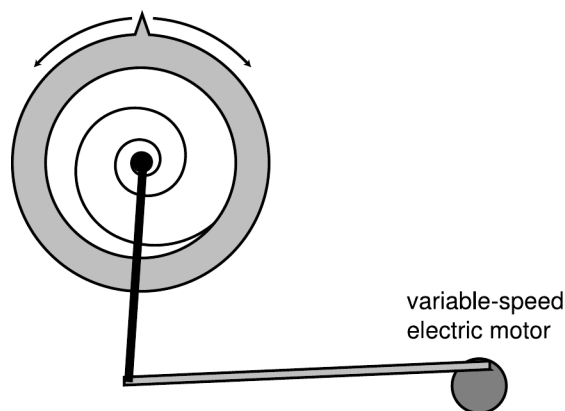
Simplified mechanical drawing of the vibrator, front view.

Apparatus

In this lab you will investigate the phenomenon of
resonance using the apparatus shown in the figure.
If the motor is stopped so that the arms are locked in

place, the metal disk can still swing clockwise and
counterclockwise because it is attached to the up-
right rod with a flexible spiral spring. A push on the
disk will result in vibrations that persist for quite
a while before the internal friction in the spring re-
duces their amplitude to an imperceptible level. This
would be an example of a free vibration, in which
energy is steadily lost in the form of heat, but no
external force pumps in energy to replace it.

Suppose instead that you initially stop the disk, but
then turn on the electric motor. There is no rigid
mechanical link to the disk, since the motor and disk
are only connected through the very flexible spiral
spring. But the motor will gently tighten and loosen
the spring, resulting in the gradual building up of a
vibration in the disk.

The pointer on the disk allows us to measure the
amplitude of the vibrations using an angular scale
(not shown). In general, we expect the energy of
a vibration to be related to the amplitude by the
approximate proportionality $E \propto A^2$, and this ap-
proximation is a pretty good one for this lab.[1]

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[1]For simple harmonic motion, the energy goes back and
forth between 100% potential and 100% kinetic. Then the
maximum potential energy is exactly proportional to A^2, and
therefore so is the kinetic energy. These statements are not all
true for the driven motion, but they are all good approxima-
tions when the driving frequency is not too far from resonance.

Observations

A Period of Free Vibrations

Start without any of the electrical stuff hooked up. Twist the disk to one side, release it, and determine its period of vibration. (Both here and at points later in the lab, you can improve your accuracy by timing ten periods and dividing the result by ten.) This is the *natural* period of the vibrations, i.e., the period with which they occur in the absence of any driving force.

B Damping

Note the coils of wire at the bottom of the disk. These are electromagnets. Their purpose is not to attract the disk magnetically (in fact the disk is made of a nonmagnetic metal) but rather to increase the amount of damping in the system. Whenever a metal is moved through a magnetic field, the electrons in the metal are made to swirl around. As they eddy like this, they undergo random collisions with atoms, causing the atoms to vibrate. Vibration of atoms is heat, so where did this heat energy come from ultimately? In our system, the only source of energy is the energy of the vibrating disk. The net effect is thus to suck energy out of the vibration and convert it into heat. Although this magnetic and electrical effect is entirely different from mechanical friction, the result is the same. Creating damping in this manner has the advantage that it can be made stronger or weaker simply by increasing or decreasing the strength of the magnetic field.

Turn off all the electrical equipment and leave it unplugged from the wall. Connect the circuit shown in the top left of the electrical diagram, consisting of a power supply to run the electromagnet. You do not yet need the power supply for driving the motor. The power supply has a built-in meter labeled "A," for "Amperes," the metric unit of electrical current. This will tell you how much electrical current is flowing through the electromagnet, giving you a numerical measure of how strong your damping is. Although this does not directly tell you the amount of damping force in units of newtons (the force depends on velocity), the force is proportional to the current.

For this part of the lab, you do not yet need any of the electrical setup for the motor (right-hand side of the figure).

Once you have everything hooked up, check with your instructor before plugging things in and turning them on. If you do the setup wrong, you could blow a fuse, which is no big deal, but a more serious goof would be to put too much current through the electromagnet, which could burn it up, permanently ruining it.

The Q of an oscillator is defined as the number of oscillations required for damping to reduce the energy of the vibrations by a factor of 535 (a definition originating from the quantity $e^{2\pi}$). As planned in your prelab, measure the Q of the system with the electromagnet turned off, then with a low current through the electromagnet, and then a higher current. There are differences among the oscillators. To compensate for this, the currents you should use should be based on which oscillator you have. These are designed to give Q values of about 8 and 15; if your actual Q's are much lower, use the spray lubricant to lubricate the bearing.

group	oscillator	low current (high Q)	high current (low Q)
1	1	180 mA	350 mA
2	2	300	490
3	3C	290	430
4	4	330	500
5	5B	400	560
6	6B	430	610
7	7B	490	610

You will be using these two current values throughout the lab. The Leybold power supply has separate knobs that set both a maximum electrical current and a maximum voltage. The easiest way to get the desired current is to *first* turn the voltage knob to zero, then set the current ("A") knob to 100%, and then turn up the voltage. The two currents listed are intended to give Q values of about 15 and 8. If you find that your Q's are much lower than this, ask your instructor for help lubricating the bearing.

For parts B and C, you do not yet need the multimeter used for measuring the speed of the motor.

C Frequency of Driven Vibration

Now connect the 24 V AC power supply to the motor. The motor has coarse (0 to 100) and fine (-6 to +6) adjustment knobs. These knobs are not calibrated in Hz, and their readings don't even correlate linearly with frequency, so to measure the frequency of the motor you need to use the stopwatch.

Set the damping current to the higher of the two values. Turn on the motor and drive the system at a frequency very different from the oscillator's natural frequency. You will notice that it takes a certain amount of time, perhaps a minute or two, for the system to settle into a steady pattern of vibration.

This is called the *steady-state response* to the driving force of the motor.

Does the system respond by vibrating at its natural frequency, at the same frequency as the motor, or at some frequency in between?

D   Resonance

For parts D and E, you will observe the response of the oscillator as a function of driving frequency and construct a graph with the square of the amplitude on the y axis and the frequency on the x axis. The reason for using the square of the amplitude is that part E is about the full width at half maximum (FWHM), and the FWHM is measured between the two points where the *energy* of the steady-state vibration equals half its maximum value. Energy is proportional to the square of the amplitude.

The purpose of part D is to determine at what frequency you obtain the strongest response.

We want to see how the results of parts D and E depend on damping, so we want graphs for both Q values. To make this part less time-consuming, you will only do the low-Q graph. A high-Q graph is given below.

resonance data by B. Crowell, Q=15
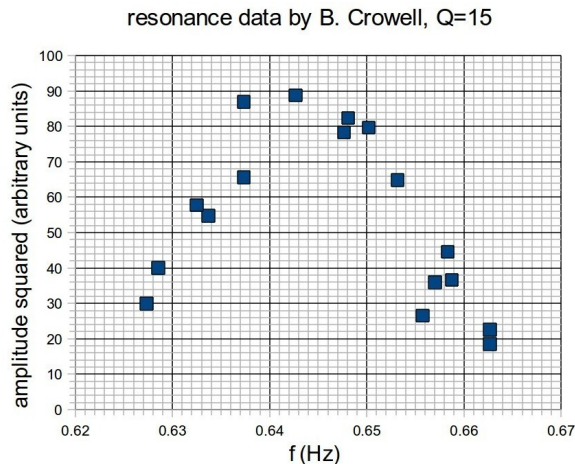
High-Q graph, taken by B. Crowell with oscillator #3, natural frequency=0.645 Hz.

When changing from one frequency to another, you have to allow time for the vibrator to reach its steady state. To make it easier to tell when the steady state is happening, it helps if you stop the vibration by hand after changing the frequency; otherwise you can get complicated patterns of motion in which the exponentially decaying motion left over from the old frequency happens on top of the new driven motion.

In order to see what's going on, you should make the graph as you go along, and let the spreadsheet program plot each point as you type it in (see appendix 4). You need to make the software understand that you intend it to graph every row that you ever enter into the spreadsheet, not just the ones that already exist when you first make the graph. To do this, use the mouse to select the first hundred rows of the two columns you're graphing, including the many blank cells below the actual data. Then make the graph. Although normally you're expected to do your analysis in this class totally independently, in this lab it's OK if you just print out multiple copies of the graph for your group.

As you're making your graph, you will see that you have certain specific places where you need to fill in data. It can be difficult to "tune in" the desired motor frequency based on the markings on the knobs. For this reason, the motor has an electrical output labeled "Ux," which gives a numerical indication of the frequency of the motor. Hook the multimeter up to it now, and set the rotary knob to a DC voltage scale as suggested in the figure, selecting the most precise scale that doesn't overload the meter. Although the readout is not calibrated in Hz, it is highly reproducible. As you take each data point, write down the reading on the meter. Then if, for example, you decide that you need to go back and get a data-point at a frequency that lies between two frequencies that you already have, you can dial in a reading on the meter that lies between the two previous readings.

E   Width of the Resonance

The goal of this part of the lab is to determine the FWHM of the resonance. Once you've located the peak of the curve in part D, the parts of the graph you need to fill in for part E are the sides.

F   Resonance Strength

Set the motor to the resonant frequency, i.e., the frequency at which you have found you obtain the strongest response. Now measure the amplitude of the vibrations you obtain with each of the two damping currents. How does the strength of the resonance depend on damping?

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise

you're just setting yourself up for failure in lab.

P1   Plan how you will determine the Q of your oscillator in part B. [Hint: Note that the energy of a vibration is proportional to the square of the amplitude.]

Analysis

Parts C, D, E, and F are all quantitative comparisons with theory. In each case, if your abstract doesn't give a theoretical number and an experimental number, you're doing something wrong. In parts D and E, you should analyze both your own graph for the low Q and the one supplied in the lab manual for the high Q.

11   Standing Waves

Apparatus

string
weights (in lab benches)
1 g slotted weights (in tray)
pulley
vibrator
paperclips
metersticks
butcher paper
scissors
weight holders
digital balances

Goals

Observe the resonant modes of vibration of a string.

Find how the speed of waves on a string depends on the tension in the string.

Introduction

The Greek philosopher Pythagoras is said to have been the first to observe that two plucked strings sounded good together when their lengths were in the proportion of two small integers. (This is assuming the strings are of the same material and under the same tension.) For instance, he thought a pleasant combination of notes was produced when one string was twice the length of the other, but that the combination was unpleasant when the ratio was, say, 1.4 to 1 (like the notes B and F). Although different combinations of notes are used in different cultures and different styles of music, there is at least some scientific justification for Pythagoras' statement. We now know that a plucked string does not just vibrate at a single frequency but simultaneously at a whole series of frequencies f_1, $2f_1$, $3f_1$,... These frequencies are called the harmonics. If one string is twice the length of the other, then its lowest harmonic is at half the frequency of the other string's, and its harmonics coincide with the odd-numbered harmonics of the other string. If the ratio is 1.4 to 1, however, then there is essentially no regular relationship between the two sets of frequencies, and many of the harmonics lie close enough in frequency to produce unpleasant beats.
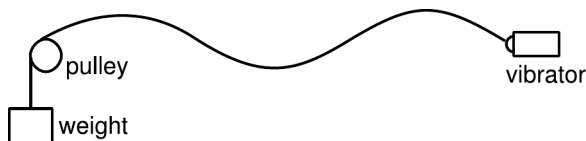
Setup

The apparatus allows you to excite vibrations at a fixed frequency of f (twice the frequency of the alternating current from the wall that runs the vibrator). Since the point of the lab is to determine a proportionality, any constant factor, such as f, can be discarded.

The tension in the string can be controlled by varying the weight.

You may find it helpful to put a strip of white butcher paper behind the black string for better visual contrast.

It's important to get the vibrator set up properly along the same line as the string, not at an angle.

If there's a loud buzzing, try moving the vibrator so that it doesn't touch the bracket holding it.

Observations

Observe as many patterns ("modes") of vibration as you can. Each mode can be labeled by N, the number of humps or half-wavelengths. You will probably not be able to observe the fundamental ($N = 1$) because it would require too much weight. In each case, you will want to fine-tune the weight to get as close as possible to the middle of the resonance, where the amplitude of vibration is at a maximum. When you're close to the peak of a resonance, an easy way to tell whether to add or remove weight is by gently pressing down or lifting up on the weights with your finger to see whether the amplitude increases or decreases.

For large values of N, you may find that you need to use a paperclip instead of the weight holder, in order to make the mass sufficiently small. Keep in mind, however, that you won't really improve the quality of your data very much by taking data for very high values of N, since the 1-gram precision with which

you can locate these resonances results in a poor relative precision compared to a small weight.

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

P1 How is the tension in the string, T, related to the mass of the hanging weight?

P2 The figure below shows the $N = 1$, 2, and 3 patterns of vibration. Suppose the length of the string is one meter. In each case, find the wavelength.
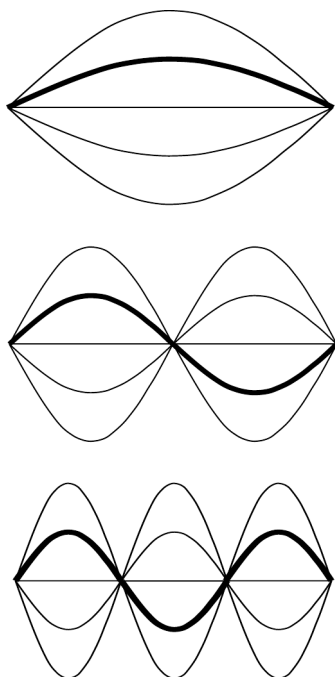
P3 Generalize your numerical results from P2 to give a general equation for λ in terms of N and L, the length of the string. Check its units, and check that it recovers the special cases done numerically in P2.

P4 How can the velocity of the waves be determined if you know the frequency, f, the length of the string, L, and the number of humps, N?

Self-Check

Do your analysis in lab.

Analysis

Use the graphing technique given in appendix 5 to see if you can find a power-law relationship between the velocity of the waves in the string and the tension in the string. Note that you can omit constant factors without affecting the exponent. (Do not just try to find the correct power law in the textbook, because besides observing the phenomenon of resonance, the point of the lab is to prove experimentally what the power-law relationship is, and to test whether this is always a good approximation in real life.)

12   Resonances of Sound

Apparatus

wave generator (Pasco PI-8127, in lab benches) 1/group
speaker (Thornton) 1/group
100 mL graduated cylinder 1/group
Linux computers with digital oscilloscope software
installed (see note above)
flexible whistling tube 1
tuning fork marked with frequency, mounted on a
wooden box 1
aluminum rod, 3/4-inch dia, about 1 m long 2
wood block 1/group
thermometer 1

Goals

Find the resonant frequencies of the air inside
a cylinder by two methods.

Measure the speeds of sound in air and in alu-
minum.

Introduction

In the womb, your first sensory experiences were of
your mother's voice, and soon after birth you learned
to distinguish the particular sounds of your parents'
voices from those of strangers. The human ear-brain
system is amazingly sophisticated in its ability to
classify vowels and consonants, recognize people's
voices, and analyze musical sound. Until the 19th-
century investigations of Helmholtz, the whole pro-
cess was completely mysterious. How could we so
easily tell a cello from a violin playing the same note?
A radio station in Chicago has a weekly contest in
which jazz fanatics are asked to identify instrumen-
talists simply by their distinctly individual timbres
— how is this possible?

Helmholtz found (using incredibly primitive nonelec-
tronic equipment) that part of the answer lay in the
relative strengths of the overtones. The psychologi-
cal sensation of pitch is related to frequency, e.g., 440
Hz is the note "A." But a saxophonist playing the
note "A" is actually producing a rich spectrum of
frequencies, including 440 Hz, 880 Hz, 1320 Hz, and
many other multiples of the lowest frequency, known
as the fundamental. The ear-brain system perceives
all these overtones as a single sound because they are
all multiples of the fundamental frequency. (The Ja-
vanese orchestra called the gamelan sounds strange
to westerners partly because the various gongs and
cymbals have overtones that are not integer multi-
ples of the fundamental.)

One of the things that would make "A" on a clarinet
sound different from "A" on a saxophone is that the
880 Hz overtone would be quite strong for the sax-
ophone, but almost entirely missing for the clarinet.
Although Helmholtz thought the relative strengths
of the overtones was the whole story when it came
to musical timbre, actually it is more complex than
that, which is why electronic synthesizers still do not
sound as good as acoustic instruments. The timbre
depends not just on the general strength of the over-
tones but on the details of how they first build up
(the attack) and how the various overtones fade in
and out slightly as the note continues.

Why do different instruments have different sound
spectra, and why, for instance, does a saxophone
have an overtone that the clarinet lacks? Many mu-
sical instruments can be analyzed physically as tubes
that have either two open ends, two closed ends, or
one open end and one closed end. The overtones
correspond to specific resonances of the air column
inside the tube. A complete treatment of the subject
is given in your textbook, but the basic principle is
that the resonant standing waves in the tube must
have an antinode (point of maximum vibration) at
any closed end of the tube, and a node (point of zero
vibration) at any open end.

Using the Wave Generator

The wave generator works like the amplifier of your
stereo, but instead of playing a CD, it produces a
sine wave whose frequency and amplitude you can
control. By connecting it to a speaker, you can con-
vert its electrical currents to sound waves, making
a pure tone. The frequency of the sine wave corre-
sponds to musical pitch, and the amplitude corre-
sponds to loudness.

Setup

Plug the speaker into the wave generator. The ba-
nana plugs go in the plugs marked OUTPUT. The
side of the banana plug converter with the tiny tab

marked GND should be the one that goes into the black output plug. Set the frequency to something audible. Turn the amplitude knob up until you hear a sound.

The wave generator and the speaker are not really designed to work together, so if you leave the volume up very high for a long time, it is possible to blow the speaker or damage the wave generator. Also, the sine waves are annoying when played continuously at loud volumes!

Preliminary Observations

Observations

This lab has three parts, A, B, and C. It is not really possible for more than one group to do part A in the same room, both because their sounds interfere with one another and because the noise becomes annoying for everyone. Your instructor will probably have three groups working on part A at one time, one group in the main room, one in the small side room, and one in the physics stockroom. Meanwhile, the other groups will be doing parts B and C.

A   Direct Measurement of Resonances by Listening

Set up the graduated cylinder so its mouth is about 3 mm from the center of the speaker grille. Find as many frequencies as possible at which the cylinder resonates. When you sweep through those frequencies, the sound becomes louder. To make sure you're really hearing a resonance of the cylinder, make sure to repeat each observation with the cylinder removed, and make sure the resonance goes away. For each resonance, take several measurements of its frequency — if you are careful, you can pin it down to within ± 10 Hz or so. You can probably speed up your search significantly by calculating approximately where you expect the resonances to be, then looking for them.

B   Electronic Measurement of Resonances of an Air Column

The resonances of the air column in a cylinder can also be excited by a stream of air flowing over an opening, as with a flute. In this part of the lab, you will excite resonances of a long, flexible plastic tube by grabbing it at one end and swinging it in a circle. The frequency of the sound will be determined electronically. Note that your analysis for these resonances will be somewhat different, since the tube is open at both ends, and it therefore has different

patterns of resonances from the graduated cylinder, which was only open at one end.

To measure the frequency, you will use a computer to analyze the sound. The Linux computers are the ones with the right hardware and software. As a warmup before attempting the actual measurements with the whistling tube, try the following. First, start up the program if nobody else has already done so. It is called "scope," and you can run it by double-clicking on its icon on the desktop. In real time, the program will monitor the sound coming into the microphone, and display a graph of loudness versus frequency. Try whistling. The frequency at which you whistled should show up as a prominent peak.

Next you need to verify that you can actually measure a known frequency and reproduce its value. Besides letting you practice using the software, this is important because I've seen some cheap computer sound input chips that produce frequencies that are off by large amounts, like 10%. Put the microphone near the wooden box that the tuning fork is mounted on and hit the tuning fork gently with the rubber mallet. To get an accurate frequency measurement, you need to zoom in on the peak. To do this, click on the peak to get an extreme close-up. To zoom back out, click on the graph again. (The Zoom In button doesn't let you get close enough, so don't use it for this purpose.) When you get the graph you want to see, you can freeze it by clicking on the Freeze/Go button.

Once you have done these warmups, you are ready to analyze the sound from the whistling tube. You only need to analyze data from one frequency, although if you're not sure which mode you produced, it may be helpful to observe the pattern of the frequencies. (If you guess wrong about which mode it was, you'll find out, because the value you extract for the speed of sound will be way off.)

C The Speed of Sound in Aluminum

The speed of sound in a solid is much faster than its speed in air. In this part of the lab, you will extract the speed of sound in aluminum from a measurement of the lowest resonant frequency of a solid aluminum rod. You will use the computer for an electronic measurement of the frequency, as in part B.

Grab the rod with two fingers exactly in the middle, hold it vertically, and tap it on the lab bench. You will hear two different notes sounding simultaneously. A quick look at their frequencies shows that they are not in a 2:1 ratio as we would expect based on our experiences with symmetric wave pat-

terns. This is because these two frequencies in the rod are actually two different types of waves. The higher note is produced by longitudinal compression waves, which means that an individual atom of aluminum is moving up and down the length of the rod. This type of wave is analogous to sound waves in air, which are also longitudinal compression waves. The lower note comes from transverse vibrations, like a vibrating guitar string. In the transverse vibrations, atoms are moving from side to side, and the rod as a whole is bending.

If you listen carefully, you can tell that the transverse vibration (the lower note) dies out quickly, but the longitudinal mode keeps going for a long time. That gives you an easy way to isolate the longitudinal mode, which is the one we're interested in; just wait for the transverse wave to die out before you freeze the graph on the computer.

Identifying the mode of vibration
The rod is symmetric, so we expect its longitudinal wave patterns to be symmetric, like those of the whistling tube. The rod is different, however, because whereas we can excite a variety of wave patterns in the tube by spinning it at different speeds, we find we only ever get one frequency from the rod by tapping it at its end: it appears that there is only one logitudinal wave pattern that can be excited strongly in the rod by this method. The problem is that we then need to infer what the pattern is.

Since you hold the rod at its center, friction should very rapidly damp out any mode of vibration that has any motion at the center. Therefore there must be a node at the center. We also know that at the ends, the rod has nothing to interact with but the air, and therefore there is essentially no way for any significant amount of wave energy to leak out; we therefore expect that waves reaching the ends have 100% of their energy reflected. Since energy is proportional to the square of amplitude, this means that a wave with unit amplitude can be reflected from the ends with an amplitude of either $R = +1$ (100% uninverted reflection) or -1 (100% inverted). In the $R = -1$ case, the reflected wave would cancel out the incident wave at the end of the rod, and we would have a node at the end, as in lab 11. In the $R = +1$ case, there would be an antinode. But when you tap the end of the rod on the floor, you are evidently exciting wave motion by moving the end, and it would not be possible to excite vibrations by this method if the vibrations had no motion at the end. We therefore conclude that the rod's pattern of vi-

bration must have a node at the center, and antinode at the ends.

There is an infinite number of possible wave patterns of this kind, but we will assume that the pattern that is excited strongly is the one with the longest wavelength, i.e., the only node is at the center, and the only antinodes are those at the ends.

If you feel like it, there are a couple of possible tests you can try to do to check whether this is the right interpretation. One is to see if you can detect any other frequencies of longitudinal vibration that are excited weakly. Another is to predict where the other nodes would be, if there were more than one, and then see if the vibration is killed by touching the rod there with your other hand; if there is a node there, touching it should have no effect.

Prelab

The point of the prelab questions is to make sure you understand what you're doing, why you're doing it, and how to avoid some common mistakes. If you don't know the answers, make sure to come to my office hours before lab and get help! Otherwise you're just setting yourself up for failure in lab.

P1 Find an equation to predict the frequencies of the resonances in parts A and B. Note that they will not be the same equations, since one tube is symmetric and the other is asymmetric. For the symmetric one, you could simply staple your prelab from lab 11 onto the back. For the asymmetric one, go through the same process that you were explicitly led through in the prelab for lab 11. This process starts with drawing the first few wave patterns. It's not necessary to define a variable N analogous to the one defined in lab 11, and in fact it's not obvious how one could define such a thing as the "number of humps" in the asymmetric case. Instead of giving a single formula with an N in it, it's fine to list expressions for the first few frequencies, showing the pattern.

Self-Check

Extract the speed of sound in air from either part A or part B, without error analysis, and make sure you get something reasonable. We don't necessarily expect it to be exactly the same as a standard value, because it depends on conditions, especially the temperature.

Analysis

Make a graph of wavelength versus period for the resonances of the graduated cylinder, check whether it looks like it theoretically should, and if so, find the speed of sound from its slope, with error bars, as discussed in appendix 4.

Analyze part B either by using the same technique (if you took data for multiple frequencies) or just by solving algebraically for the speed of sound.

The effective length of the cylinder in part A should be increased by 0.4 times its diameter to account for the small amount of air beyond the end that also vibrates. For part B, where the whistling tube is open at both ends, you should add 0.8 times its diameter.

When estimating error bars from part B, you may be tempted to say that it must be perfectly accurate, since it's being done by a computer. Not so! The graph only has a certain frequency resolution, and in addition, the peak may be a little ragged.

Extract the speed of sound in aluminum from your data in part C, including error bars.

Appendix 1: Format of Lab Writeups

Lab reports must be three pages or less, not counting your raw data. The format should be as follows:

Title

Raw data — *Keep actual observations separate from what you later did with them.*
These are the results of the measurements you take down during the lab, hence they come first. Write your raw data directly in your lab book; don't write them on scratch paper and recopy them later. Don't use pencil. The point is to separate facts from opinions, observations from inferences.

Procedure — *Did you have to create your own methods for getting some of the raw data?*
Do not copy down the procedure from the manual. In this section, you only need to explain any methods you had to come up with on your own, or cases where the methods suggested in the handout didn't work and you had to do something different. Don't write anything here unless you think I will really care and want to change how we do the lab in the future. In most cases this section can be totally blank. Do not discuss how you did your calculations here, just how you got your raw data.

Abstract — *What did you find out? Why is it important?*
The "abstract" of a scientific paper is a *short* paragraph at the top that summarizes the experiment's results in a few sentences.

Many of our labs are comparisons of theory and experiment. The abstract for such a lab needs to say whether you think the experiment was consistent with theory, or not consistent with theory. If your results deviated from the ideal equations, don't be afraid to say so. After all, this is real life, and many of the equations we learn are only approximations, or are only valid in certain circumstances. However, (1) if you simply mess up, it is your responsibility to realize it in lab and do it again, right; (2) you will never get exact agreement with theory, because measurements are not perfectly exact — the important issue is whether your results agree with theory to roughly within the error bars.

The abstract is not a statement of what you hoped to find out. It's a statement of what you *did* find out. It's like the brief statement at the beginning of a debate: "The U.S. should have free trade with China." It's not this: "In this debate, we will discuss whether the U.S. should have free trade with China."

If this is a lab that has just one important numerical result (or maybe two or three of them), put them in your abstract, with error bars where appropriate. There should normally be no more than two to four numbers here. Do not recapitulate your raw data here — this is for your final results.

If you're presenting a final result with error bars, make sure that the number of significant figures is consistent with your error bars. For example, if you write a result as 323.54 ± 6 m/s, that's wrong. Your error bars say that you could be off by 6 in the ones' place, so the 5 in the tenths' place and the four in the hundredths' place are completely meaningless.

If you're presenting a number in scientific notation, with error bars, don't do it like this

$$1.234 \times 10^{-89} \text{ m/s} \pm 3 \times 10^{-92} \text{ m/s} \quad ,$$

do it like this

$$(1.234 \pm 0.003) \times 10^{-89} \text{ m/s} \quad ,$$

so that we can see easily which digit of the result the error bars apply to.

Calculations and Reasoning — *Convince me of what you claimed in your abstract.*
Often this section consists of nothing more than the calculations that you started during lab. If those calculations are clear enough to understand, and there is nothing else of interest to explain, then it is not necessary to write up a separate narrative of your analysis here. If you have a long series of similar calculations, you may just show one as a sample. If your prelab involved deriving equations that you will need, repeat them here without the derivation.

In some labs, you will need to go into some detail here by giving logical arguments to convince me that the statements you made in the abstract follow logically from your data. Continuing the debate metaphor, if your abstract said the U.S. should have free trade with China, this is the rest of the debate, where you convince me, based on data and logic, that we should have free trade.

Appendix 2: Basic Error Analysis

No measurement is perfectly exact.

One of the most common misconceptions about science is that science is "exact." It is always a struggle to get beginning science students to believe that no measurement is perfectly correct. They tend to think that if a measurement is a little off from the "true" result, it must be because of a mistake — if a pro had done it, it would have been right on the mark. Not true!

What scientists can do is to estimate just how far off they might be. This type of estimate is called an error bar, and is expressed with the ± symbol, read "plus or minus." For instance, if I measure my dog's weight to be 52 ± 2 pounds, I am saying that my best estimate of the weight is 52 pounds, and I think I could be off by roughly 2 pounds either way. The term "error bar" comes from the conventional way of representing this range of uncertainty of a measurement on a graph, but the term is also used when no graph is involved.

Some very good scientific work results in measurements that nevertheless have large error bars. For instance, the best measurement of the age of the universe is now 15 ± 5 billion years. That may not seem like wonderful precision, but the people who did the measurement knew what they were doing. It's just that the only available techniques for determining the age of the universe are inherently poor.

Even when the techniques for measurement are very precise, there are still error bars. For instance, electrons act like little magnets, and the strength of a very weak magnet such as an individual electron is customarily measured in units called Bohr magnetons. Even though the magnetic strength of an electron is one of the most precisely measured quantities ever, the best experimental value still has error bars: $1.0011596524 \pm 0.0000000002$ Bohr magnetons.

There are several reasons why it is important in scientific work to come up with a numerical estimate of your error bars. If the point of your experiment is to test whether the result comes out as predicted by a theory, you know there will always be some disagreement, even if the theory is absolutely right. You need to know whether the measurement is reasonably consistent with the theory, or whether the discrepancy is too great to be explained by the limitations of the measuring devices.

Another important reason for stating results with error bars is that other people may use your measurement for purposes you could not have anticipated. If they are to use your result intelligently, they need to have some idea of how accurate it was.

Error bars are not absolute limits.

Error bars are not absolute limits. The true value may lie outside the error bars. If I got a better scale I might find that the dog's weight is 51.3 ± 0.1 pounds, inside my original error bars, but it's also possible that the better result would be 48.7 ± 0.1 pounds. Since there's always some chance of being off by a somewhat more than your error bars, or even a lot more than your error bars, there is no point in being extremely conservative in an effort to make absolutely sure the true value lies within your stated range. When a scientist states a measurement with error bars, she is not saying "If the true value is outside this range, I deserve to be drummed out of the profession." If that was the case, then every scientist would give ridiculously inflated error bars to avoid having her career ended by one fluke out of hundreds of published results. What scientists are communicating to each other with error bars is a typical amount by which they might be off, not an upper limit.

The important thing is therefore to define error bars in a standard way, so that different people's statements can be compared on the same footing. By convention, it is usually assumed that people estimate their error bars so that about two times out of three, their range will include the true value (or the results of a later, more accurate measurement with an improved technique).

Random and systematic errors.

Suppose you measure the length of a sofa with a tape measure as well as you can, reading it off to the nearest millimeter. If you repeat the measurement again, you will get a different answer. (This is assuming that you don't allow yourself to be psychologically biased to repeat your previous answer, and that 1 mm is about the limit of how well you can see.) If you kept on repeating the measurement,

you might get a list of values that looked like this:

| 203.1 cm | 203.4 | 202.8 | 203.3 | 203.2 |
| 203.4 | | 203.1 | 202.9 | 202.9 | 203.1 |

Variations of this type are called random errors, because the result is different every time you do the measurement.

The effects of random errors can be minimized by averaging together many measurements. Some of the measurements included in the average are too high, and some are too low, so the average tends to be better than any individual measurement. The more measurements you average in, the more precise the average is. The average of the above measurements is 203.1 cm. Averaging together many measurements cannot completely eliminate the random errors, but it can reduce them.

On the other hand, what if the tape measure was a little bit stretched out, so that your measurements always tended to come out too low by 0.3 cm? That would be an example of a systematic error. Since the systematic error is the same every time, averaging didn't help us to get rid of it. You probably had no easy way of finding out exactly the amount of stretching, so you just had to suspect that there might a systematic error due to stretching of the tape measure.

Some scientific writers make a distinction between the terms "accuracy" and "precision." A precise measurement is one with small random errors, while an accurate measurement is one that is actually close to the true result, having both small random errors and small systematic errors. Personally, I find the distinction is made more clearly with the more memorable terms "random error" and "systematic error."

The $\pm$ sign used with error bars normally implies that random errors are being referred to, since random errors could be either positive or negative, whereas systematic errors would always be in the same direction.

The goal of error analysis

Very seldom does the final result of an experiment come directly off of a clock, ruler, gauge or meter. It is much more common to have raw data consisting of direct measurements, and then calculations based on the raw data that lead to a final result. As an example, if you want to measure your car's gas mileage, your raw data would be the number of gallons of gas consumed and the number of miles you went. You would then do a calculation, dividing
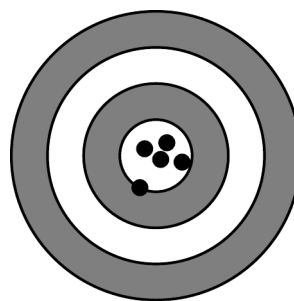
small random errors,
small systematic error
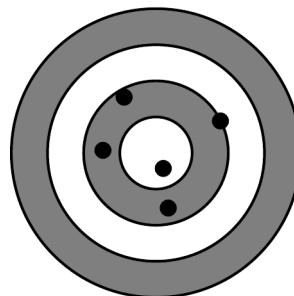
large random errors,
small systematic error
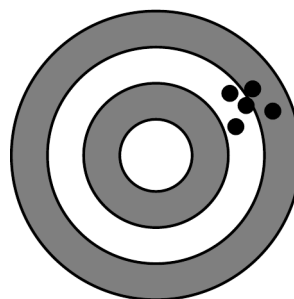
small random errors,
large systematic error

miles by gallons, to get your final result. When you communicate your result to someone else, they are completely uninterested in how accurately you measured the number of miles and how accurately you measured the gallons. They simply want to know how accurate your final result was. Was it 22 ± 2 mi/gal, or 22.137 ± 0.002 mi/gal?

Of course the accuracy of the final result is ultimately based on and limited by the accuracy of your raw data. If you are off by 0.2 gallons in your measurement of the amount of gasoline, then that amount of error will have an effect on your final result. We say that the errors in the raw data "propagate" through the calculations. When you are requested to do "error analysis" in a lab writeup, that means that you

are to use the techniques explained below to determine the error bars on your final result. There are two sets of techniques you'll need to learn:

techniques for finding the accuracy of your raw data

techniques for using the error bars on your raw data to infer error bars on your final result

Estimating random errors in raw data

We now examine three possible techniques for estimating random errors in your original measurements, illustrating them with the measurement of the length of the sofa.

Method #1: Guess

If you're measuring the length of the sofa with a metric tape measure, then you can probably make a reasonable guess as to the precision of your measurements. Since the smallest division on the tape measure is one millimeter, and one millimeter is also near the limit of your ability to see, you know you won't be doing better than $\pm$ 1 mm, or 0.1 cm. Making allowances for errors in getting tape measure straight and so on, we might estimate our random errors to be a couple of millimeters.

Guessing is fine sometimes, but there are at least two ways that it can get you in trouble. One is that students sometimes have too much faith in a measuring device just because it looks fancy. They think that a digital balance must be perfectly accurate, since unlike a low-tech balance with sliding weights on it, it comes up with its result without any involvement by the user. That is incorrect. No measurement is perfectly accurate, and if the digital balance only displays an answer that goes down to tenths of a gram, then there is no way the random errors are any smaller than about a tenth of a gram.

Another way to mess up is to try to guess the error bars on a piece of raw data when you really don't have enough information to make an intelligent estimate. For instance, if you are measuring the range of a rifle, you might shoot it and measure how far the bullet went to the nearest centimeter, concluding that your random errors were only $\pm$1 cm. In reality, however, its range might vary randomly by fifty meters, depending on all kinds of random factors you don't know about. In this type of situation, you're better off using some other method of estimating your random errors.

Method #2: Repeated Measurements and the Two-Thirds Rule

If you take repeated measurements of the same thing, then the amount of variation among the numbers can tell you how big the random errors were. This approach has an advantage over guessing your random errors, since it automatically takes into account all the sources of random error, even ones you didn't know were present.

Roughly speaking, the measurements of the length of the sofa were mostly within a few mm of the average, so that's about how big the random errors were. But let's make sure we are stating our error bars according to the convention that the true result will fall within our range of errors about two times out of three. Of course we don't know the "true" result, but if we sort out our list of measurements in order, we can get a pretty reasonable estimate of our error bars by taking half the range covered by the middle two thirds of the list. Sorting out our list of ten measurements of the sofa, we have

| 202.8 cm | 202.9 | 202.9 | 203.1 | 203.1 |
| 203.1 | 203.2 | 203.3 | 203.4 | 203.4 |

Two thirds of ten is about 6, and the range covered by the middle six measurements is 203.3 cm - 202.9 cm, or 0.4 cm. Half that is 0.2 cm, so we'd estimate our error bars as ±0.2 cm. The average of the measurements is 203.1 cm, so your result would be stated as 203.1 ± 0.2 cm.

One common mistake when estimating random errors by repeated measurements is to round off all your measurements so that they all come out the same, and then conclude that the error bars were zero. For instance, if we'd done some overenthusiastic rounding of our measurements on the sofa, rounding them all off to the nearest cm, every single number on the list would have been 203 cm. That wouldn't mean that our random errors were zero! The same can happen with digital instruments that automatically round off for you. A digital balance might give results rounded off to the nearest tenth of a gram, and you may find that by putting the same object on the balance again and again, you always get the same answer. That doesn't mean it's perfectly precise. Its precision is no better than about ±0.1 g.

Method #3: Repeated Measurements and the Standard Deviation

The most widely accepted method for measuring error bars is called the standard deviation. Here's how the method works, using the sofa example again.

(1) Take the average of the measurements.

$$\text{average} = 203.1 \text{ cm}$$

(2) Find the difference, or "deviation," of each measurement from the average.

−0.3 cm	−0.2	−0.2	0.0	0.0
0.0	0.1	0.1	0.3	0.3

(3) Take the square of each deviation.

0.09 cm^2	0.04	0.04	0.00	0.00
0.00	0.01	0.01	0.09	0.09

(4) Average together all the squared deviations.

$$\text{average} = 0.04 \text{ cm}^2$$

(5) Take the square root. This is the standard deviation.

$$\text{standard deviation} = 0.2 \text{ cm}$$

If we're using the symbol x for the length of the couch, then the result for the length of the couch would be stated as $x = 203.1 \pm 0.2$ cm, or $x = 203.1$ cm and $\sigma_x = 0.2$ cm. Since the Greek letter sigma (σ) is used as a symbol for the standard deviation, a standard deviation is often referred to as "a sigma."

Step (3) may seem somewhat mysterious. Why not just skip it? Well, if you just went straight from step (2) to step (4), taking a plain old average of the deviations, you would find that the average is zero! The positive and negative deviations always cancel out exactly. Of course, you could just take absolute values instead of squaring the deviations. The main advantage of doing it the way I've outlined above are that it is a standard method, so people will know how you got the answer. (Another advantage is that the standard deviation as I've described it has certain nice mathematical properties.)

A common mistake when using the standard deviation technique is to take too few measurements. For instance, someone might take only two measurements of the length of the sofa, and get 203.4 cm and 203.4 cm. They would then infer a standard deviation of zero, which would be unrealistically small because the two measurements happened to come out the same.

In the following material, I'll use the term "standard deviation" as a synonym for "error bar," but that does not imply that you must always use the standard deviation method rather than the guessing method or the 2/3 rule.

There is a utility on the class's web page for calculating standard deviations.

Probability of deviations

You can see that although 0.2 cm is a good figure for the typical size of the deviations of the measurements of the length of the sofa from the average, some of the deviations are bigger and some are smaller. Experience has shown that the following probability estimates tend to hold true for how frequently deviations of various sizes occur:
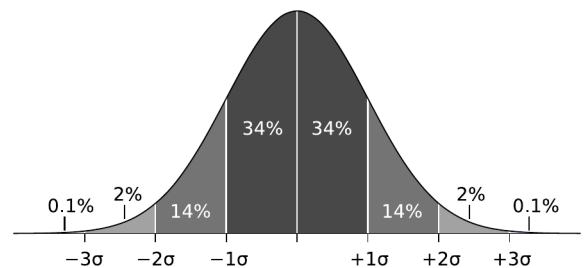
> 1 standard deviation about 1 times out of 3

> 2 standard deviations about 1 time out of 20

> 3 standard deviations about 1 in 500

> 4 standard deviations about 1 in 16,000

> 5 standard deviations about 1 in 1,700,000

The probability of various sizes of deviations, shown graphically. Areas under the bell curve correspond to probabilities. For example, the probability that the measurement will deviate from the truth by less than one standard deviation ($\pm 1\sigma$) is about $34 \times 2 = 68\%$, or about 2 out of 3. (J. Kemp, P. Strandmark, Wikipedia.)

Example: How significant?
In 1999, astronomers Webb et al. claimed to have found evidence that the strength of electrical forces in the ancient universe, soon after the big bang, was slightly weaker than it is today. If correct, this would be the first example ever discovered in which the laws of physics changed over time. The difference was very small, 5.7 ± 1.0 parts per million, but still highly statistically significant. Dividing, we get $(5.7 - 0)/1.0 = 5.7$ for the number of standard deviations by which their measurement was different from the expected result of zero. Looking at the table above, we see that if the true value really was zero, the chances of this happening would be less than one in a million. In general, five standard deviations ("five sigma") is considered the gold standard for statistical significance.

This is an example of how we test a hypothesis statistically, find a probability, and interpret the probability. The probability we find is the probability that our results

would differ this much from the hypothesis, if the hypothesis was true. It's not the probability that the hypothesis is true or false, nor is it the probability that our experiment is right or wrong.

However, there is a twist to this story that shows how statistics always have to be taken with a grain of salt. In 2004, Chand et al. redid the measurement by a more precise technique, and found that the change was 0.6 ± 0.6 parts per million. This is only one standard deviation away from the expected value of 0, which should be interpreted as being statistically consistent with zero. If you measure something, and you think you know what the result is supposed to be theoretically, then one standard deviation is the amount you typically *expect* to be off by — that's why it's called the "standard" deviation. Moreover, the Chand result is wildly statistically inconsistent with the Webb result (see the example on page 55), which means that one experiment or the other is a mistake. Most likely Webb at al. underestimated their random errors, or perhaps there were systematic errors in their experiment that they didn't realize were there.

Precision of an average

We decided that the standard deviation of our measurements of the length of the couch was 0.2 cm, i.e., the precision of each individual measurement was about 0.2 cm. But I told you that the average, 203.1 cm, was more precise than any individual measurement. How precise is the average? The answer is that the standard deviation of the average equals

$$\frac{\text{standard deviation of one measurement}}{\sqrt{\text{number of measurements}}} \; .$$

(An example on page 54 gives the reasoning that leads to the square root.) That means that you can theoretically measure anything to any desired precision, simply by averaging together enough measurements. In reality, no matter how small you make your random error, you can't get rid of systematic errors by averaging, so after a while it becomes pointless to take any more measurements.

Appendix 3: Propagation of Errors

Propagation of the error from a single variable

In the previous appendix we looked at techniques for estimating the random errors of raw data, but now we need to know how to evaluate the effects of those random errors on a final result calculated from the raw data. For instance, suppose you are given a cube made of some unknown material, and you are asked to determine its density. Density is defined as $\rho = m/v$ (ρ is the Greek letter "rho"), and the volume of a cube with edges of length b is $v = b^3$, so the formula

$$\rho = m/b^3$$

will give you the density if you measure the cube's mass and the length of its sides. Suppose you measure the mass very accurately as $m = 1.658 \pm 0.003$ g, but you know $b = 0.85 \pm 0.06$ cm with only two digits of precision. Your best value for ρ is $1.658 \text{ g}/(0.85 \text{ cm})^3 = 2.7 \text{ g/cm}^3$.

How can you figure out how precise this value for ρ is? We've already made sure not to keep more than twosignificant figures for ρ, since the less accurate piece of raw data had only two significant figures. We expect the last significant figure to be somewhat uncertain, but we don't yet know how uncertain. A simple method for this type of situation is simply to change the raw data by one sigma, recalculate the result, and see how much of a change occurred. In this example, we add 0.06 cm to b for comparison.

$b = 0.85$ cm   gave $\rho = 2.7 \text{ g/cm}^3$
$b = 0.91$ cm   gives $\rho = 2.2 \text{ g/cm}^3$

The resulting change in the density was 0.5 g/cm^3, so that is our estimate for how much it could have been off by:

$$\rho = 2.7 \pm 0.5 \text{ g/cm}^3 \qquad .$$

Propagation of the error from several variables

What about the more general case in which no one piece of raw data is clearly the main source of error? For instance, suppose we get a more accurate measurement of the edge of the cube, $b = 0.851 \pm 0.001$ cm. In percentage terms, the accuracies of m and b are roughly comparable, so both can cause sig-

nificant errors in the density. The following more general method can be applied in such cases:

(1) Change one of the raw measurements, say m, by one standard deviation, and see by how much the final result, ρ, changes. Use the symbol Q_m for the absolute value of that change.

$m = 1.658$ g   gave $\rho = 2.690 \text{ g/cm}^3$
$m = 1.661$ g   gives $\rho = 2.695 \text{ g/cm}^3$

$Q_m = $ change in $\rho = 0.005 \text{ g/cm}^3$

(2) Repeat step (1) for the other raw measurements.

$b = 0.851$ cm   gave $\rho = 2.690 \text{ g/cm}^3$
$b = 0.852$ cm   gives $\rho = 2.681 \text{ g/cm}^3$

$Q_b = $ change in $\rho = 0.009 \text{ g/cm}^3$

(3) The error bars on ρ are given by the formula

$$\sigma_\rho = \sqrt{Q_m^2 + Q_b^2} \qquad ,$$

yielding $\sigma_\rho = 0.01 \text{ g/cm}^3$. Intuitively, the idea here is that if our result could be off by an amount Q_m because of an error in m, and by Q_b because of b, then if the two errors were in the same direction, we might by off by roughly $|Q_m| + |Q_b|$. However, it's equally likely that the two errors would be in opposite directions, and at least partially cancel. The expression $\sqrt{Q_m^2 + Q_b^2}$ gives an answer that's smaller than $Q_m + Q_b$, representing the fact that the cancellation might happen.

The final result is $\rho = 2.69 \pm 0.01 \text{ g/cm}^3$.

Example: An average
On page 52 I claimed that averaging a bunch of measurements reduces the error bars by the square root of the number of measurements. We can now see that this is a special case of propagation of errors.

For example, suppose Alice measures the circumference c of a guinea pig's waist to be 10 cm, Using the guess method, she estimates that her error bars are about ± 1 cm (worse than the normal normal ~ 1 mm error bars for a tape measure, because the guinea pig was squirming). Bob then measures the same thing, and gets 12 cm. The average is computed as

$$c = \frac{A + B}{2} \qquad ,$$

where *A* is Alice's measurement, and *B* is Bob's, giving 11 cm. If Alice had been off by one standard deviation (1 cm), it would have changed the average by 0.5

cm, so we have $Q_A = 0.5$ cm, and likewise $Q_B = 0.5$ cm. Combining these, we find $\sigma_c = \sqrt{Q_A^2 + Q_B^2} = 0.7$ cm, which is simply $(1.0 \text{ cm})/\sqrt{2}$. The final result is $c = (11.0 \pm 0.7)$ cm. (This violates the usual rule for significant figures, which is that the final result should have no more sig figs than the least precise piece of data that went into the calculation. That's okay, because the sig fig rules are just a quick and dirty way of doing propagation of errors. We've done real propagation of errors in this example, and it turns out that the error is in the first decimal place, so the 0 in that place is entitled to hold its head high as a real sig fig, albeit a relatively imprecise one with an uncertainty of ± 7.)

Example: The difference between two measurements
In the example on page 51, we saw that two groups of scientists measured the same thing, and the results were $W = 5.7 \pm 1.0$ for Webb et al. and $C = 0.6 \pm 0.6$ for Chand et al. It's of interest to know whether the difference between their two results is small enough to be explained by random errors, or so big that it couldn't possibly have happened by chance, indicating that someone messed up. The figure shows each group's results, with error bars, on the number line. We see that the two sets of error bars don't overlap with one another, but error bars are not absolute limits, so it's perfectly possible to have non-overlapping error bars by chance, but the gap between the error bars is very large compared to the error bars themselves, so it looks implausible that the results could be statistically consistent with one another. I've tried to suggest this visually with the shading underneath the data-points.
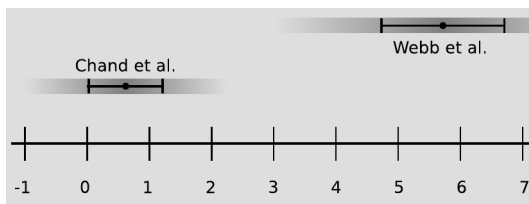
To get a sharper statistical test, we can calculate the difference d between the two results,

$$d = W - C \quad ,$$

which is 5.1. Since the operation is simply the subtraction of the two numbers, an error in either input just causes an error in the output that is of the same size. Therefore we have $Q_W = 1.0$ and $Q_C = 0.6$, resulting in $\sigma_d = \sqrt{Q_W^2 + Q_C^2} = 1.2$. We find that the difference between the two results is $d = 5.1 \pm 1.2$, which differs from zero by $5.1/1.2 \approx 4$ standard deviations. Looking at the table on page 51, we see that the chances that d would be this big by chance are extremely small, less than about one in ten thousand. We can conclude to a high level of statistical confidence that the two groups' measurements are inconsistent with one another, and that one group is simply wrong.

Appendix 4: Graphing

Review of Graphing

Many of your analyses will involve making graphs. A graph can be an efficient way of presenting data visually, assuming you include all the information needed by the reader to interpret it. That means labeling the axes and indicating the units in parentheses, as in the example. A title is also helpful. Make sure that distances along the axes correctly represent the differences in the quantity being plotted. In the example, it would not have been correct to space the points evenly in the horizontal direction, because they were not actually measured at equally spaced points in time.
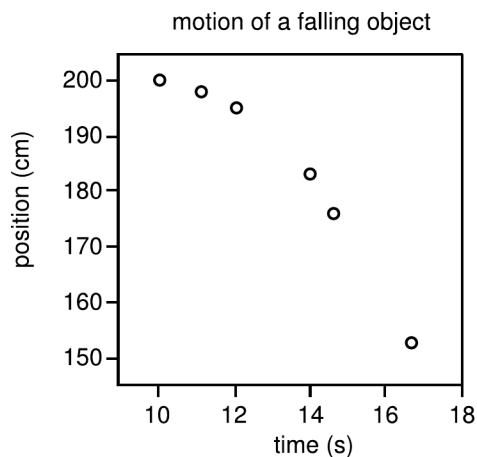
Graphing on a Computer

Making graphs by hand in your lab notebook is fine, but in some cases you may find it saves you time to do graphs on a computer. For computer graphing, I recommend LibreOffice, which is free, open-source software. It's installed on the computers in rooms 416 and 418. Because LibreOffice is free, you can download it and put it on your own computer at home without paying money. If you already know Excel, it's very similar — you almost can't tell it's a different program.

Here's a brief rundown on using LibreOffice:

> On Windows, go to the Start menu and choose All Programs, LibreOffice, and LibreOffice Calc. On Linux, do Applications, Office, OpenOffice, Spreadsheet.

Type in your x values in the first column, and your y values in the second column. For scientific notation, do, e.g., 5.2e-7 to represent 5.2×10^{-7}.

Select those two columns using the mouse.

From the Insert menu, do Object:Chart.

When it offers you various styles of graphs to choose from, choose the icon that shows a scatter plot, with dots on it (XY Chart).

Adjust the scales so the actual data on the plot is as big as possible, eliminating wasted space. To do this, double-click on the graph so that it's surrounded by a gray border. Then do Format, Axis, X Axis or Y Axis, Scale.

If you want error bars on your graph you can either draw them in by hand or put them in a separate column of your spreadsheet and doing Insert, Y Error Bars, Cell Range. Under Parameters, check "Same value for both." Click on the icon, and then use the mouse in the spreadsheet to select the cells containing the error bars.

Fitting a Straight Line to a Graph by Hand

Often in this course you will end up graphing some data points, fitting a straight line through them with a ruler, and extracting the slope.
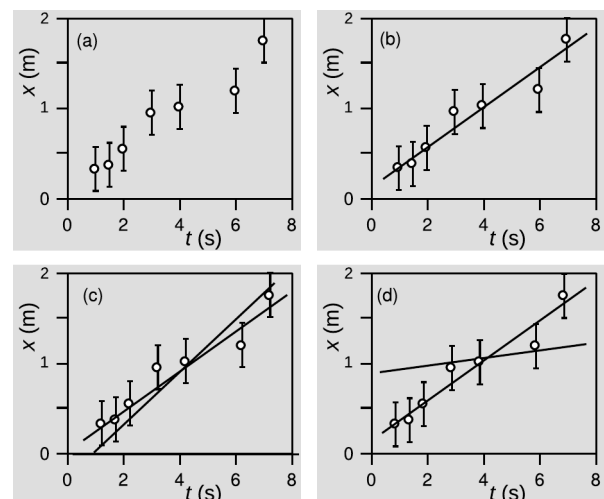
In this example, panel (a) shows the data, with error bars on each data point. Panel (b) shows a best fit, drawn by eye with a ruler. The slope of this best fit line is 100 cm/s. Note that the slope should be extracted from the line itself, not from two data points. The line is more reliable than any pair of individual data points.

In panel (c), a "worst believable fit" line has been drawn, which is as different in slope as possible from the best fit, while still pretty much staying consistent the data (going through or close to most of the error bars). Its slope is 60 cm/s. We can therefore estimate that the precision of our slope is +40 cm/s.

There is a tendency when drawing a "worst believable fit" line to draw instead an "unbelievably crazy fit" line, as in panel (d). The line in panel (d), with a very small slope, is just not believable compared to the data — it is several standard deviations away from most of the data points.

Fitting a Straight Line to a Graph on a Computer

It's also possible to fit a straight line to a graph using computer software such as LibreOffice.

To do this, first double-click on the graph so that a gray border shows up around it. Then right-click on a data-point, and a menu pops up. Choose Insert Trend Line.[1] choose Linear, and check the box for Show equation.

How accurate is your slope? A method for getting error bars on the slope is to artificially change one of your data points to reflect your estimate of how much it could have been off, and then redo the fit and find the new slope. The change in the slope tells you the error in the slope that results from the error in this data-point. You can then repeat this for the other points and proceed as in appendix 3.

An alternative method is to use the LINEST function that is available in many spreadsheet programs. For a description, see `tinyurl.com/ya7wmdft`. Create the following formula in one cell of your spreadsheet: =Linest(y-values,x-value, True.True). Then, in excel, you need to press alt+ctrl+enter. In google sheets, press enter. A table with two columns and five rows will appear. The first number in the first column is the slope of the graph, and the second

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[1] "Trend line" is scientifically illiterate terminology that originates from Microsoft Office, which LibreOffice slavishly copies. If you don't want to come off as an ignoramus, call it a "fit" or "line of best fit."

number in the first column is the error in the slope.

In some cases, such as the absolute zero lab and the photoelectric effect lab, it's very hard to tell how accurate your raw data are *a priori*; in these labs, you can use the typical amount of deviation of the points from the line as an estimate of their accuracy.

Comparing Theory and Experiment

Figures (e) through (h) are examples of how we would compare theory and experiment on a graph. The convention is that theory is a line and experiment is points; this is because the theory is usually a prediction in the form of an equation, which can in principle be evaluated at infinitely many points, filling in all the gaps. One way to accomplish this with computer software is to graph both theory and experiment as points, but then print out the graph and draw a smooth curve through the theoretical points by hand.
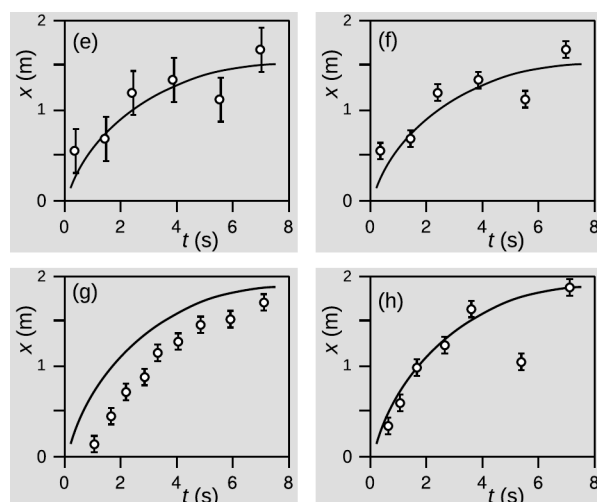
The point here is usually to compare theory and experiment, and arrive at a yes/no answer as to whether they agree. In (e), the theoretical curve goes through the error bars on four out of six of the data points. This is about what we expect statistically, since the probability of being within one standard deviation of the truth is about 2/3 for a standard bell curve. Given these data, we would conclude that theory and experiment agreed.

In graph (f), the points are exactly the same as in (e), but the conclusion is the opposite. The error bars are smaller, too small to explain the observed discrepancies between theory and experiment. The theoretical curve only goes through the error bars on two of the six points, and this is quite a bit less than

we would expect statistically.

Graph (g) also shows disagreement between theory and experiment, but now we have a clear systematic error. In (h), the fifth data point looks like a mistake. Ideally you would notice during lab that something had gone wrong, and go back and check whether you could reproduce the result.

Appendix 5: Finding Power Laws from Data

For many people, it is hard to imagine how scientists originally came up with all the equations that can now be found in textbooks. This appendix explains one method for finding equations to describe data from an experiment.

Linear and nonlinear relationships

When two variables x and y are related by an equation of the form

$$y = cx \qquad ,$$

where c is a constant (does not depend on x or y), we say that a linear relationship exists between x and y. As an example, a harp has many strings of different lengths which are all of the same thickness and made of the same material. If the mass of a string is m and its length is L, then the equation

$$m = cL$$

will hold, where c is the mass per unit length, with units of kg/m. Many quantities in the physical world are instead related in a nonlinear fashion, i.e., the relationship does not fit the above definition of linearity. For instance, the mass of a steel ball bearing is related to its diameter by an equation of the form

$$m = cd^3 \qquad ,$$

where c is the mass per unit volume, or density, of steel. Doubling the diameter does not double the mass, it increases it by a factor of eight.

Power laws

Both examples above are of the general mathematical form

$$y = cx^p \qquad ,$$

which is known as a power law. In the case of a linear relationship, $p = 1$. Consider the (made-up) experimental data shown in the table.

	h=height of rodent at the shoulder (cm)	f=food eaten per day (g)
shrew	1	3
rat	10	300
capybara	100	30,000

It's fairly easy to figure out what's going on just by staring at the numbers a little. Every time you increase the height of the animal by a factor of 10, its food consumption goes up by a factor of 100. This implies that f must be proportional to the square of h, or, displaying the proportionality constant $k = 3$ explicitly,

$$f = 3h^2 \qquad .$$

Use of logarithms

Now we have found $c = 3$ and $p = 2$ by inspection, but that would be much more difficult to do if these weren't all round numbers. A more generally applicable method to use when you suspect a power-law relationship is to take logarithms of both variables. It doesn't matter at all what base you use, as long as you use the same base for both variables. Since the data above were increasing by powers of 10, we'll use logarithms to the base 10, but personally I usually just use natural logs for this kind of thing.

	$\log_{10} h$	$\log_{10} f$
shrew	0	0.48
rat	1	2.48
capybara	2	4.48

This is a big improvement, because differences are so much simpler to work mentally with than ratios. The difference between each successive value of h is 1, while f increases by 2 units each time. The fact that the logs of the $f's$ increase twice as quickly is the same as saying that f is proportional to the square of h.

Log-log plots

Even better, the logarithms can be interpreted visually using a graph, as shown on the next page. The slope of this type of log-log graph gives the power p. Although it is also possible to extract the proportionality constant, c, from such a graph, the proportionality constant is usually much less interesting than p. For instance, we would suspect that if $p = 2$ for rodents, then it might also equal 2 for frogs or ants. Also, p would be the same regardless of what units we used to measure the variables. The constant c, however, would be different if we used different units, and would also probably be different for other types of animals.
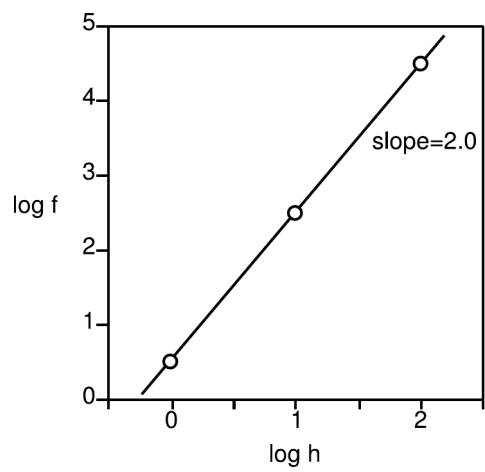
Appendix 6: Comment Codes for Lab Writeups

A. General

a1. Don't write numbers without units. (25% off)

a2. If something is wrong, cross it out. Don't make me guess which version to grade.

a3. Your writeup is too long. The length limit is 3 pages, not including raw data.

a4. If your writeup includes printouts, staple them in sideways with a single staple.

a5. See appendix 1 for the format of lab writeups.

a6. Don't state speculation as a firm conclusion.

a7. Leave more space for me to write comments.

a8. Cut unnecessary words. Use active voice. Write in a simple, direct style.

a9. Don't write walls of text. Use paragraph breaks.

a10. Cut any sentence that doesn't carry information.

a11. This paragraph needs a topic sentence.

a12. Express this as an equation.

a13. Don't present details unless you've already made it clear why we would care. Don't write slavishly in chronological order.

a14. The first sentence of any piece of writing must make an implicit promise that the remainder will interest the reader.

B. Raw data

b1. Don't mix raw data with calculations. (25% off)

b2. Write raw data in pen, directly in the notebook.

b3. This isn't raw data. This is a summary or copy.

C. Procedure

c1. Don't repeat the lab manual.

c2. Don't write anything about your procedure unless it's something truly original that you think I would be interested in knowing about, or I wouldn't be able to understand your writeup without it.

D. Abstract – see appendix 1

d1. Your abstract is too long.

d2. Don't recap raw data in your abstract.

d3. Don't describe calculations in your abstract.

d4. The only numbers that should be in your abstract are important final results that support your conclusion or that constitute the purpose of the lab.

d5. Your abstract needs to include numerical results that support your conclusions.

d6. Give error bars in your abstract.

d7. Where is your abstract?

d8. Your abstract is for results. This isn't a result of your experiment.

d9. This isn't important enough to go in your abstract.

d10. What was the point of the lab, and why would anyone care?

d11. Don't just give results. Interpret them.

d12. We knew this before you did the lab.

d13. This lab was a quantitative test. Restating it qualitatively isn't interesting.

d14. This lab is a comparison of theory and experiment. Did they agree, or not?

d15. Your results don't support your conclusions. Write about what really happened, not what you wanted to happen.

d16. One observation can never prove a general rule.

E. Error analysis – see appendices 2 and 3

e1. A standard deviation only measures error if it comes from numbers that were supposed to be the same, e.g., repeated measurements of the same thing.

e2. In propagation of errors, don't do both high and low. See appendix 3.

e3. In propagation of errors, only change one variable at a time. See appendix 3.

e4. Don't round severely when calculating Q's. Your Q's are just measuring your rounding errors.

e5. A Q is the amount by which the output of the calculation changes, not its inputs.

e6. A Q is a change in the result, not the

result itself.

e7. Use your error bars in forming your conclusions. Otherwise what was the point of calculating them?

e8. Give a probabilistic interpretation, as in the examples at the end of appendix 2.

e9. You're interpreting this probability incorrectly. It's the probability that your results would have differed this much from the hypothesis, if the hypothesis were true.

e10. % errors are useless. Teachers have you do them if you don't know about real error analysis.

e11. If random errors are included in your propagation of errors, listing them here verbally is pointless.

e12. Don't speculate about systematic errors without investigating them. Estimate their possible size. Would they produce an effect in the right direction?

G. Graphing – see appendix 4

g1. Label the axes to show what variables are being graphed and what their units are, e.g., x (km).

g2. Your graph should be bigger.

g3. If graphing by hand, do it on graph paper.

g4. Choose an appropriate scale for your graph, so that the data are not squished down. Don't just accept the default from the software if it's wrong. See app. 4 for how to do this using Libre Office.

g5. "Dot to dot" style is wrong in a scientific graph.

g6. The independent variable (the one you control directly) goes on the x axis, and the dependent variable on the y. Or: cause on x, effect on y.

g7. On a scientific graph, use dots to show data, a line or curve for theory or a fit to the data.

g8. "Trend line" is scientifically illiterate. It's called a line of best fit.

S. Calculations and sig figs

s1. *Think* about the sizes of numbers and whether they make sense. This number doesn't make sense.

s2. Where did this number come from?

s3. This number has too many sig figs (e.g., more than the number of sig figs in the raw data).

s4. Don't round off severely for sig figs at intermediate steps. Rounding errors can accumulate.

s5. You're wasting your time by writing down many non-significant figures.

s6. Your result has too many sig figs. The error bars show that you don't have this much precision.

s7. The Calculations and Reasoning section usually just consists of the calculations you've already written.You don't need to write a separate narrative.

s8. Put your calculator in scientific notation mode.