

Practice Exam 5 — Fundamentals of Calculus, ch. 1-10

1 Evaluate the following. Simplify your answers as much as possible.

(a) $\int x^7 dx$

(b) $\int 3x^{-2} dx$

(c) $\int (x^3 + 2x^4) dx$

(d) $\int e^{2x} dx$

(e) $\int \sqrt{x}(x + x^2) dx$

2 Evaluate the following. All letters other than the variable of integration are constants. Simplify your answers as much as possible.

(a) $\int A(1 - q)^{37} dq$

(b) $\int h^3 \cos(kh^4) dh$

(c) $\int z\sqrt{a + bz} dz$

(d) $\int N \cdot (1/2)^{t/\tau} dt$

(e) $\int \frac{A df}{B + Cf}$

(f) $\int \frac{1 + 2\beta x}{x + \beta x^2} dx$

3 Evaluate the following. Simplify your answers as much as possible.

(a) $\int_{-1}^1 (x + x^3 + x^5 + x^7 + x^9) dx$

(b) $\int_0^1 (1 - x)^2 dx$

(c) $\int_0^{2\pi} (\sin x + 3 \cos x) dx$

(d) $\int_1^7 (x^{-2} + x^{-1}) dx$

(e) $\int_0^{\sqrt{2}} \frac{dx}{\sqrt{1 - x^2/4}}$

4 Determine whether the following are correct or incorrect, and explain how you know. For the ones that are incorrect, fix the right-hand side to make it correct.

(a) $\int e^{7x} dx = e^{7x} + c$

(b) $\int \ln x dx = x \ln x - x + c$

(c) $\int (x \sin x - \cos x) dx = x \cos x + c$

5 (a) Show that

$$\int \frac{dx}{x^2 + x} = -\ln(1 + 1/x) + c.$$

(b) Evaluate

$$\int \frac{dx}{x^2 + Cx},$$

where C is a nonzero constant.

Answer to problem 1

- (a) $\int x^7 dx = (1/8)x^8 + c$ (integral of a power)
- (b) $\int 3x^{-2} dx = -3x^{-1} + c$ (integral of a power, linearity for the constant 3)
- (c) $\int (x^3 + 2x^4) dx = (1/4)x^4 + (2/5)x^5 + c$ (power, linearity)
- (d) $\int e^{2x} dx = (1/2)e^{2x} + c$ (similar to e^x , guess and check to get the 1/2)
- (e) $\int \sqrt{x}(x + x^2) dx = \int (x^{3/2} + x^{5/2}) dx = (2/5)x^{5/2} + (2/7)x^{7/2} + c$

Answer to problem 2

- (a) $\int A(1 - q)^{37} dq = -(1/38)A(1 - q)^{38} + c$ (substitute $u = 1 - q$, or just guess and check)
- (b) Let $u = kh^4$. $\int h^3 \cos(kh^4) dh = (1/4k) \sin(kh^4) + c$
- (c) Let $u = a + bz$, $dz = du/b$. $\int z\sqrt{a + bz} dz = b^{-2} \int (u - a)\sqrt{u} du = 2b^{-2} \left[(1/5)(a + bz)^{5/2} - (a/3)(a + bz)^{3/2} \right] + c$
- (d) $\int N \cdot (1/2)^{t/\tau} dt = N \int \exp[(-\ln 2/\tau)t] dt$. Let $k = -\ln 2/\tau$. The integral then becomes $N \int e^{kt} dt = (N/k)e^{kt} + c = (-\tau N/\ln 2)(1/2)^{t/\tau} + c$.
- (e) Let $u = B + Cf$, $df = du/C$. $\int \frac{A df}{B + Cf} = (A/C) \int du/u = A/C \ln u + k = (A/C) \ln(B + Cf) + k$
- (f) Let $u = x + \beta x^2$. $\int \frac{1 + 2\beta x}{x + \beta x^2} dx = \int du/u = \ln(x + \beta x^2) + c$

Answer to problem 3

- (a) The function being integrated is odd, so since the integral runs from -1 to 1 , the result is 0 by symmetry.
- (b) With a change of variable, this is the same as $\int_0^1 u^2 du = 1/3$.
- (c) The integral runs over a full cycle of the sine and cosine, so the result is zero.
- (d) The indefinite integral is $-x^{-1} + \ln x$, so the definite integral works out to be $6/7 + \ln 7$.
- (e) With the substitution $u = x/2$, we have

$$\begin{aligned} \int_0^{\sqrt{2}} \frac{dx}{\sqrt{1 - x^2/4}} &= \int_0^{1/\sqrt{2}} \frac{2du}{\sqrt{1 - u^2}} \\ &= 2 \int_0^{1/\sqrt{2}} \frac{du}{\sqrt{1 - u^2}} \\ &= 2 \sin^{-1} u \Big|_0^{1/\sqrt{2}} \\ &= \pi/2 \end{aligned}$$

Answer to problem 4

- (a) Differentiating e^{7x} gives $7e^{7x}$, not e^{7x} , the factor of 7 coming from the chain rule. The integral is wrong as written. To make it correct, we need to fix the factor of 7, so the correct answer is $(1/7)e^{7x} + c$.

(b) Differentiation gives $\ln x + x/x - 1 = \ln x$, which is correct.

(c) The derivative is $\cos x - x \sin x$, which has the wrong sign over all. The correct integral is $-x \cos x + c$.

Answer to problem 5

(a) Differentiation gives:

$$\frac{d}{dx} (-\ln(1 + 1/x) + c) = -\frac{1}{1 + 1/x} \cdot (-x^{-2}) = \frac{1}{x^2 + x}$$

(b) To make this look like the form in part a, we do the substitution $u = x/C$, which results in

$$\begin{aligned} \int \frac{dx}{x^2 + Cx} &= \int \frac{C du}{C^2u^2 + c^2u} \\ &= \frac{1}{C} \int \frac{du}{u^2 + u} \\ &= -\frac{1}{C} \ln(1 + 1/u) + c \\ &= -\frac{1}{C} \ln(1 + C/x) + c \end{aligned}$$