Practice Exam 2 for Physics 205/210 — Light and Matter, ch. 0-9

1 *This question is for Physics 205 only.* Sometimes restaurants give you a drinking straw that comes in a paper wrapper. You can tear off one end of the wrapper and blow so that it shoots the wrapper off like shooting a gun. Suppose that the straw is 19 cm long, the wrapper has a mass of 105 mg, the force from your breath is constant, and the wrapper is launched with a speed of 3.4 m/s. Find the force.

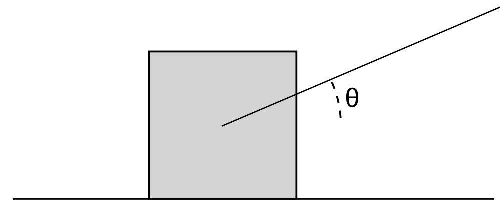
2 *This question is for Physics 210 only.* A playground has a metal bunny on a spring with a saddle so that little kids can bounce up and down on it. The motion is given by $x = A\cos(\omega t)e^{-bt}$. The mass of the child plus the bunny is m. Find the total force acting on them as a function of time. You may wish to check (not for credit) that your units come out right.

3 (a) In the Bohr model of the hydrogen atom, an electron travels in a circular orbit around a proton. If the proton's force on the electron is purely attractive (i.e., parallel to the line between them), then why doesn't the electron slow down, so that the atom collapses? (b) If observers in different frames of reference look at the motion of the same object, they will give different values for its velocity. Since Newton's second law depends on acceleration, and acceleration is defined in terms of velocity, how can the object obey Newton's second law in both frames?

4 A sailor is driving a cart across the deck of an aircraft carrier. The aircraft carrier, A, is sailing at 22 m/s relative to the ocean, O, in the direction 17 degrees counterclockwise from east. The cart, C, is moving *relative to the ship* at 14 m/s, 11 degrees *clockwise* from east. The sailor tosses a pack of cigarettes, P, to her crewmate, who is also riding on the cart. While in flight, the cigarettes happen to be exactly at rest relative to the ocean. (This entire problem deals only with motion in the horizontal plane.) Find the direction and speed at which the cigarettes, P, are moving *relative to the cart*, C. Give the direction as an angle, and include a diagram showing how the direction is defined. Hint: Since P is at rest with respect to O, you can treat them as the same object; you really have 3 objects, not 4.

5 The pads on dogs' feet have a low coefficient of friction on common types of household floors, which is why when you see a dog try to run around a corner in a house, often its feet will slip. Suppose that a dog and a human are both running around a corner along an arc with the same radius, but the coefficient of friction is greater by a factor of 2.3 for the human's shoes. Find the ratio of the maximum speeds at which they can run without slipping.

6 The figure shows a box of mass m being dragged across a surface using a rope at angle θ. This is similar to an experiment you did in class, except that in class we tried to make $\theta = 0$.



Question 6

(a) Analyze the forces in which the box participates.

Copy the following table header onto your paper.

force acting on box			equal and opposite force involved in Newton's 3rd law		
type of force	object exerting the force	direction	type of force	object exerting the force	direction

(b) If the tension needed in order to drag the box at constant velocity is measured to be T, find the coefficient of kinetic friction μ_k.
(c) Check that the units of your answer make sense.
(d) Check the dependence of your answer on T. This means that you should first decide how the result *should* physically depend on T (whether raising T should make μ_k bigger, or smaller), and then figure out how your answer depends on T mathematically, and compare the two.

More practice:
Solve ch. 8, # 6 (about a skier) twice, using each of the following two methods.
(1) Pick a coordinate system (it's easiest if you pick one tilted to coincide with the slope) and do analytic addition of force vectors, as in sec. 8.3, example 7 (pushing a broom). To determine the components of the vectors, use the technique described in sec. 8.3, example 6 (a layback) and practiced in ch. 8, #18.
(2) Draw a geometrical diagram showing graphical addition of the force vectors, as in sec. 8.3, example 5 (pushing a block up a ramp). Do geometry and trig on that triangle.

Solve ch. 8, # 4, 5, 14, and 16.

Answer to problem 1
Because you've had good training, you're going to solve this algebraically and only plug in numbers at the end. Let L be the length of the straw, m the mass of the wrapper, v the launch speed, and F the force. The relevant constant-acceleration equation gives $v^2 = 2aL$, and Newton's second law is $a = F/m$. Eliminating a, we find

$$\begin{aligned} F &= mv^2/2L \\ &= \frac{(1.05\times 10^{-4}\ \text{kg})(3.4\ \text{m/s})^2}{2(0.19\ \text{m})} \\ &= 3.2\times 10^{-3}\ \text{N}. \end{aligned}$$

(When I wrote this problem, I tried to use fairly realistic numbers as inputs, and to me, it's surprising that the force is so small, since it feels like a significant physical effort to blow out the air.)

Answer to problem 2

$$\begin{aligned} \frac{dx}{dt} &= -A\omega\sin(\omega t)e^{-bt} - Ab\cos(\omega t)e^{-bt} \\ \frac{d^2x}{dt^2} &= -A\omega^2\cos(\omega t)e^{-bt} + Ab\omega\sin(\omega t)e^{-bt} + Ab\omega\sin(\omega t)e^{-bt} + Ab^2\cos(\omega t)e^{-bt} \\ &= Ae^{-bt}[(-\omega^2+b^2)\cos(\omega t) + 2b\omega\sin(\omega t)] \\ F &= ma \\ &= mAe^{-bt}[(-\omega^2+b^2)\cos(\omega t) + 2b\omega\sin(\omega t)] \end{aligned}$$

Unit check: If the given equation for x is to make sense, then A must have units of meters, and both ω and b must be in s^{-1}. The answer then has units of $\text{kg}\cdot\text{m}[\text{s}^{-2}+\text{s}^{-2}]$, which does come out to be newtons.

Answer to problem 3
(a) Only an inward force is required for circular motion at constant speed. One might think that a forward force would be needed in order to maintain the forward motion, but that's an Aristotelian preconception. Motion doesn't naturally slow down.
(b) The observers disagree on velocities, but they agree on accelerations. For simplicity, let's consider the case of constant acceleration in one dimension, so that $a = \Delta v/\Delta t$. Switching frames of reference adds a constant onto all velocities, but the Δv is the *change* in velocity, so it's unaffected by the addition of this constant.

Answer to problem 4
We start by converting all angles into angles measured counterclockwise from east. This changes the $11\,°$ to $-11\,°$.

$$\begin{aligned} \mathbf{v}_{PC} &= \mathbf{v}_{OC} \\ &= \mathbf{v}_{OA} + \mathbf{v}_{AC} \\ &= -\mathbf{v}_{AO} - \mathbf{v}_{CA} \\ v_{PC,x} &= -v_{AO,x} - v_{CA,x} \\ &= -(22\ \text{m/s})\cos 17\,° - (14\ \text{m/s})\cos(-11\,°) \\ &= -21.0\ \text{m/s} - 13.7\ \text{m/s} \qquad \text{[These make sense.]} \\ &= -34.8\ \text{m/s} \end{aligned}$$

$$
\begin{aligned}
v_{PC,y} &= -v_{AO,y} - v_{CA,y} \\
&= -(22\text{ m/s})\sin 17° - (14\text{ m/s})\sin(-11°) \\
&= -6.4\text{ m/s} + 2.7\text{ m/s} \qquad \text{[These make sense.]} \\
&= -3.8\text{ m/s} \\
|\mathbf{v}_{PC}| &= \sqrt{v_{PC,x}^2 + v_{PC,y}^2} \\
&= 35\text{ m/s} \\
\theta &= \tan^{-1}(v_{PC,y}/v_{PC,x}) \\
&= \{6°, 186°\}
\end{aligned}
$$

Drawing a sketch, we can see that the correct arctangent is 186°, which is measured counterclockwise from east.

Answer to problem 5
We have $a = v^2/r$ for circular motion, and r is fixed, so $v \propto \sqrt{a} \propto \sqrt{\mu}$. Therefore $v_2/v_1 = \sqrt{\mu_2/\mu_1} = \sqrt{2.3} = 1.5$.

Answer to problem 6
(a)

force acting on box			*equal and opposite force involved in Newton's 3rd law*		
type of force	*direction*	*object exerting the force*	*type*	*direction*	*object exerting it*
gravity	down	earth	gravity	up	box
normal	up	surface	normal	down	box
kinetic friction	left	surface	kinetic friction	right	box
normal?	up and right	rope	normal?	down and left	box

(b) Since the motion is at constant velocity, the acceleration is zero, and the total force on the block is zero. Let the normal force between the surface and the box be F_N. Let positive x be to the right, and positive y up. The total force in the x direction is zero:

$$T\cos\theta - F_N\mu_k = 0.$$

The total force in the y direction is also zero:

$$F_N - mg + T\sin\theta = 0.$$

Eliminating F_N and solving for μ_s gives

$$\mu_k = \frac{\cos\theta}{mg/T - \sin\theta}.$$

(c) The ratio mg/T is unitless, so it makes sense to subtract a unitless sine from it. Both the numerator and the denominator are unitless, so the right-hand side is unitless. That makes sense, because μ_k is unitless based on its definition.
(d) Physically, if it takes a lot of tension to make the box slide, that would indicate a large coefficient of friction. Mathematically, raising T makes mg/T smaller, which makes the denominator smaller, and therefore the right-hand side gets bigger. The physical and mathematical behaviors match up.