Practice Exam 4 for Physics 205 — LM, ch. 0-15

1 A baseball pitcher exerts a force of 50 N over a distance of 1.4 m. The ball's mass is 145 g. Find the speed of the pitch.

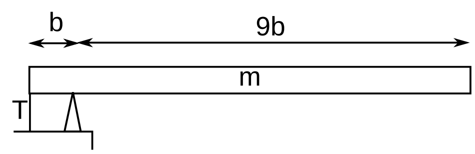
2 An object of mass m is moving to the east at speed v. A second object, also of mass m, is moving to the south at speed $2v$. (a) Find the direction and magnitude of the total momentum.
(b) Find the total kinetic energy.
(c) Suppose that the masses were to collide and stick together. What would be conserved? Momentum? Kinetic energy? Both? Neither?

3 A ballet dancer spins clockwise on one toe, decelerates, and then starts spinning counterclockwise. Discuss the changes in her momentum, kinetic energy, and angular momentum.

4 Aliens come to our solar system and gather together a mountainous pile of rock from the asteroid belt. Then they glue this rock together into two huge uniform spheres, each of mass m and radius r. They put these two spheres side by side, touching at point P, and start them rotating about P at such a speed that each sphere is in a circular orbit of radius just slightly more than r, so that there is a tiny bit of empty space at P. At P, they place a microscopic shrine to Audrey Hepburn. (a) Find the period T of the orbit. (b) Check that your answer to part a has units that make sense.

5 Objects A and B collide head-on. Compared to A, B has double the mass and 1/3 the kinetic energy. If they collide and stick, which object "wins," i.e., do they end up going in A's original direction of motion, or B's?

6 The figure shows a uniform horizontal object of mass m, such as a diving board, supported by a fulcrum at a point one tenth of the way from one end. It is prevented from toppling by the tension T in a cable on the left. (a) Find the tension in terms of variables that would be directly known to the designer (variables that would be known without the need for any calculation). (b) Check that the units of your answer make sense.



Question 6

Answer to problem 1
The kinetic energy K equals the work Fd done on the ball. We then have $v = \sqrt{2K/m} = \sqrt{2Fd/m} =$ 31 m/s.

Answer to problem 2
(a) Let positive x be east, and positive y north. Then the first object has momentum $\mathbf{p}_1 = mv\hat{\mathbf{x}}$, while the second one has $\mathbf{p}_2 = -2mv\hat{\mathbf{y}}$. The total momentum is $\mathbf{P} = mv(\hat{\mathbf{x}} - 2\hat{\mathbf{y}})$. The magnitude of this vector is $|\mathbf{P}| = mv\sqrt{1^2+2^2} = \sqrt{5}mv$. Its direction, expressed as an angle counterclockwise from east, is $\tan^{-1}(-2/1) = \{-63\,°, 117\,°\}$. The first arctangent is the correct one: 63 ° clockwise from east.
(b) The individual kinetic energies are $(1/2)mv^2$ and $2mv^2$, and the total is $(5/2)mv^2$.
(c) Momentum is conserved. Energy is also conserved, but what is conserved is the total energy, not the kinetic energy. In this type of sticking collision, it is not possible for the kinetic energy to be conserved, i.e., some energy is transformed into other forms, such as heat, sound, or deformation of the objects.

Answer to problem 3
Her momentum is always zero. Roughly speaking, this is because momentum is a vector, and momentum from any element of mass on one side of her body cancels pairwise with the corresponding mass on the opposite side. A little more rigorously, her center of mass doesn't move, so her momentum must be zero.

Kinetic energy is a scalar, and it's always positive, because it depends on the square of the velocity. Therefore there is no cancellation of kinetic energy. Her kinetic energy decreases, becomes zero momentarily, and then increases again.

Let's describe clockwise rotation as positive. Her angular momentum starts out at zero, becomes positive, returns to zero, and finally becomes negative.

Answer to problem 4
(a) For the motion of one of the asteroids, Newton's second law gives $a = F/m$, or $v^2/r = Gm/(2r)^2$. Eliminating the velocity $v = 2\pi r/T$ gives $T = 4\pi r^{3/2}(Gm)^{-1/2}$.
(b) The units are

$$\mathrm{m}^{3/2}\left(\frac{\mathrm{N{\cdot}m^2kg}}{\mathrm{kg}^2}\right)^{-1/2} = \mathrm{m}^{3/2}\left(\frac{\mathrm{kg{\cdot}m/s^2{\cdot}m^2}}{\mathrm{kg}}\right)^{-1/2} = \mathrm{m}^{3/2}(\mathrm{m}^3/\mathrm{s}^2)^{-1/2} = \mathrm{s}.$$

Answer to problem 5
Momentum is conserved, so after the collision, they must be moving in the direction of the initial total momentum vector. Solving for momentum in terms of mass and kinetic energy, we have $p = \sqrt{2mK}$, or $p \propto \sqrt{mK}$. The ratio of their momenta is therefore

$$\frac{p_B}{p_A} = \sqrt{\frac{m_B}{m_A}\frac{K_B}{K_A}} = \sqrt{2/3},$$

so B's momentum is smaller, and A wins.

Answer to problem 6
The forces are all vertical. In principle we would expect to approach this by writing down an equation stating that the total force along the vertical axis is zero, and another equation stating that the total torque is zero. We would then have two equations in two unknowns: the tension in the cable and the force made by the fulcrum.

But it's quite a bit simpler if we take the axis to be the fulcrum and only use the torque equation. We then have $\tau_{\text{cable}} + \tau_{\text{grav}} = 0$. Let counterclockwise torques be positive. Then the torque made by the cable is Tb. The force of gravity is treated as if it acted at the center of mass, at a distance $4b$ from the fulcrum, so the gravitational torque is $-4bmg$. Then $Tb - 4bmg = 0$, so $T = 4mg$.