Practice Exam 2 for Physics 206/211 — Light and Matter, ch. 0-26

Useful Data

gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Coulomb constant	$k = 8.99 \times 10^9$ J·m/C^2 or N·m^2/C^2
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
quantum of charge	$e = 1.60 \times 10^{-19}$ C
speed of light	$c = 3.00 \times 10^8$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
absolute zero	$-273.15\,°$C
speed of sound in air	340 m/s
mass of electron	9.11×10^{-31} kg

Some Magnetic Fields
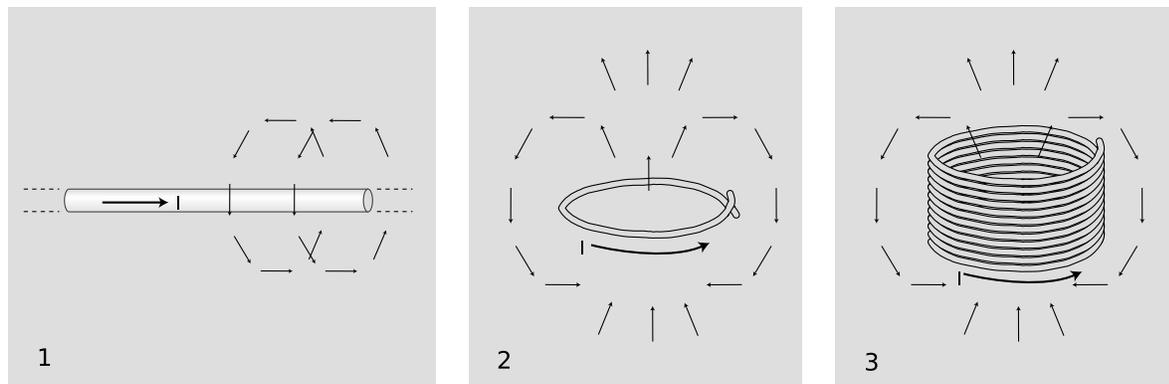
1. Field created by a long, straight wire carrying current I:

$$B = \frac{k}{c^2} \cdot \frac{2I}{r}$$

Here r is the distance from the center of the wire. The field vectors trace circles in planes perpendicular to the wire, going clockwise when viewed from along the direction of the current.

2. Field created by a single circular loop of current:
The field vectors form a dipole-like pattern, coming through the loop and back around on the outside. Each oval path traced out by the field vectors appears clockwise if viewed from along the direction the current is going when it punches through it. There is no simple equation for a field at an arbitrary point in space, but for a point lying *along the central axis* perpendicular to the loop, the field is

$$B = \frac{k}{c^2} \cdot 2\pi I b^2 \left(b^2 + z^2\right)^{-3/2} \quad ,$$

where b is the radius of the loop and z is the distance of the point from the plane of the loop.

3. Field created by a solenoid (cylindrical coil):
The field pattern is similar to that of a single loop, but for a long solenoid the paths of the field vectors become very straight on the inside of the coil and on the outside immediately next to the coil. For a sufficiently long solenoid, the interior field also becomes very nearly uniform, with a magnitude of

$$B = \frac{k}{c^2} \cdot 4\pi I N/\ell \quad ,$$

where N is the number of turns of wire and ℓ is the length of the solenoid. The field near the mouths or outside the coil is not constant, and is more difficult to calculate. For a long solenoid, the exterior field is much smaller than the interior field.

1 (a) Human vision is most sensitive to light with a wavelength of about 550 nm, which is a yellowish color typical of sunlight. Find the frequency of light that has this wavelength.
(b) Suppose that we want to take a certain light wave and make its energy 17 times greater. By what factor do we have to increase its electric field? Its magnetic field?

2 Let a wire carry current $I > 0$ along the x axis, flowing the positive x direction. A second wire, this one lying along the y axis, carries $2I$ in the negative y direction. (The wires have to make a slight detour to avoid one another, but we assume that this is negligible. Both wires extend infinitely far in both directions.) Suppose that at a certain point (x, y) in the x-y plane, there is zero magnetic field. Relate x to y, simplifying the equation as much as possible.

3 Diagram 1 shows a solenoid made by wrapping wire around a toilet paper roll. It is powered by a 1.5-V battery, and produces a field of 10 mT at point P. Point P is directly across from the center of the solenoid. The arrows show the direction of the current in each circuit.

(a) Indicate with an arrow the direction of the magnetic field at point P in diagram 1. (If you think you might get the direction of the field reversed, just give it your best guess and continue with the problem. You will get nearly full credit if you do everything right except that all your fields are in the opposite direction compared to what they should be.)
(b) An electron is at point P, moving to the right. Find the direction of the force acting on it.
(c) In diagram 2, a second, identical solenoid is added, and 9 V batteries are used to power both solenoids. What is the magnitude of the magnetic field at point P in diagram 2?
(d) Indicate with an arrow the direction of the magnetic field in diagram 2.
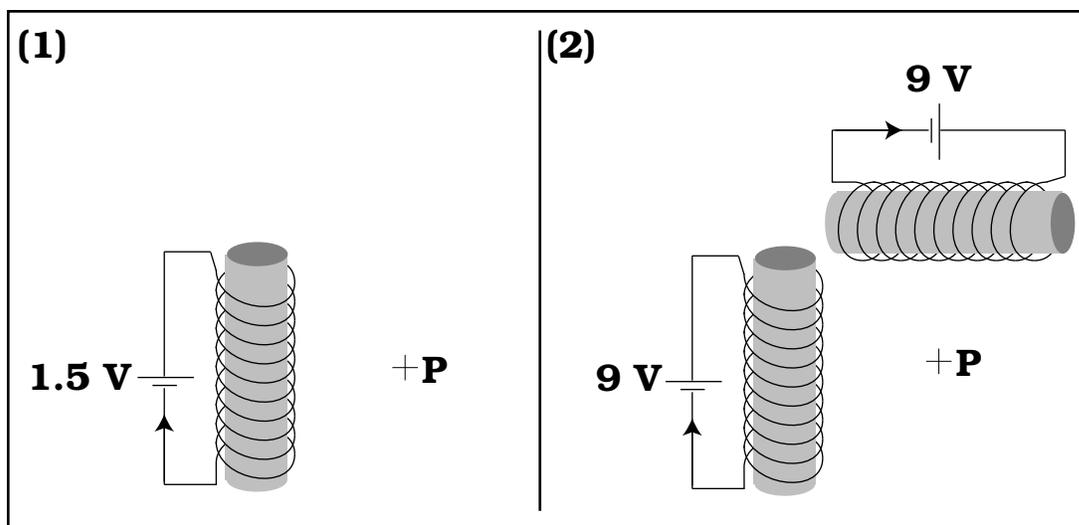
Question 3

4 The figure shows a graph of position versus time, depicted in some inertial frame of reference. The units on the x axis are light-seconds; a light-second is the distance traveled by light in one second. A and B are events, and C is a graph of the motion of an observer named Chuck.
(a) Find Chuck's velocity in this frame of reference.
(b) What is Chuck's velocity in his own frame of reference?
(c) Is Chuck accelerating, or is he moving inertially, like an object described by Newton's first law? Explain how you know.
(d) A radio beep is emitted from B. Find the time when Chuck receives it.
(e) In some other frame of reference, events A and B are simultaneous. Find the velocity of that frame of reference relative to this one.
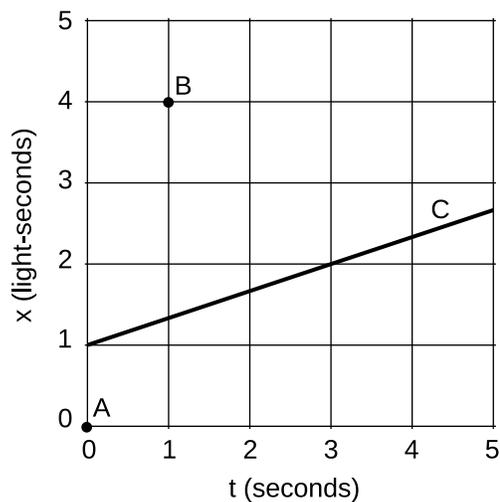
Question 4

5 (a) How do we know that electrons exist?
(b) If forced to hold a powerful radioactive source in the palm of your bare hand, would you choose one that emitted alpha, beta, or gamma radiation?

Answer to problem 1

(a)
$$f = \frac{v}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{5.50 \times 10^{-7} \text{ m}} = 5.5 \times 10^{14} \text{ Hz}$$

(b) The electric and magnetic fields in an electromagnetic wave are in fixed proportion, so whatever we do to one of the fields, we have to do the same to the other. The energy density of each field is proportional to the square of the field, so each field has to be increased by a factor of $\sqrt{17} = 4.1$.

Answer to problem 2

The first diagram on the front page of the exam shows the direction of the field surrounding a wire, and below it is given an equation that tells us that $B \propto I/r$. To make the fields cancel, we need them to be in opposite directions, which happens in quadrants II and IV. We also need them to be equal in magnitude, which means that we have to be twice as far from the vertical wire as from the horizontal one. Therefore the points where the fields cancel are given by the condition $y = -x/2$.

Answer to problem 3

(a) The current is counterclockwise as seen from above, so the orientation is the same as in the figure on the first page where the formula for the interior field is given. Therefore the field at P is down.

(b) For a velocity pointing to the right and a magnetic field pointing down, the right-hand rule gives a force pointing into the page. Since the electron is negatively charged, the force is out of the page.

(c) Instead of the 1.5 V battery, we now have a 9 V battery driving each coil. Therefore the current in each coil is 6 times greater than before, and each coil's contribution to the magnetic field is 60 mT rather than 10 mT. Referring to the diagram, the second solenoid's contribution to the field is to the left. Let positive x be to the right and positive y up. Then the original solenoid gives $B_{1x} = 0$, $B_{1y} = -60$ mT, and the new solenoid gives $B_{2x} = -60$ mT, $B_{2y} = 0$. The total field has $B_x = -60$ mT and $B_y = -60$ mT, so its magnitude is $\sqrt{B_x^2 + B_y^2} = 84$ mT.

(d) The x and y components are negative and equal to each other, so the field points down and to the left at a 45 degree angle.

Answer to problem 4

(a) Chuck moves 1 light-second in 3 seconds, so his velocity is 1/3 in natural units, i.e., $c/3$.

(b) Chuck's velocity is zero in his own frame of reference.

(c) He's moving inertially. We can tell this because his graph is a line.

(d) The beep moves at c, which is depicted on this graph as a line with a slope of ± 1. The part of the expanding wavefront with slope -1 is the one that will reach Chuck. Chuck receives it at $t = 3$ seconds, which is the point where the beep's graph intersects Chuck's.

(e) When we do a Lorentz transformation, the x and t axes rotate like a pair of scissors towards the 45-degree diagonal. To make A and B simultaneous, we have to rotate the x axis enough to make it run through B, which means that as depicted on this graph, it would go 1 unit over and 4 units up. The matching behavior for the t axis would be that it would go 4 units over and 1 up. This is a slope of 1/4, so the velocity of that frame of reference would have to be 1/4, i.e., $c/4$.

Answer to problem 5

(a) In experiments with vacuum tubes, we can produce beams of particles. By subjecting these particles to electric and magnetic fields, we can determine their charge-to-mass ratio q/m. This ratio is very large compared to the values for ionized atoms, which suggests either than q is very large, or m is a mass much smaller than that of an atom.

(b) I'd choose the alpha source, because the alphas wouldn't be able to penetrate the dead layer of skin (epidermis). (The betas might also be stopped by the epidermis, but it would depend on their energy and on how thick the epidermis is.)