Practice Exam 3 for Physics 206 — through ch. 31

1 You have access to a converging mirror, a diverging mirror, a converging lens, and a diverging lens. You want to form a virtual image of something. Of the four optical devices you have, which ones can be used, and under what conditions?

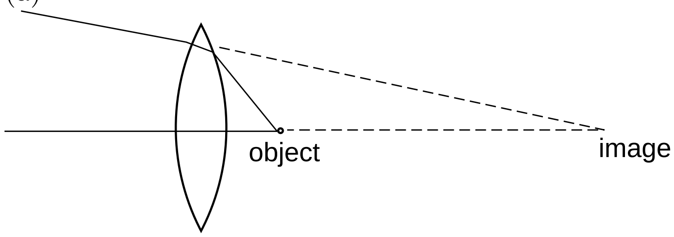
2 A converging lens is used to produce a virtual image. (a) Draw a ray diagram and indicate the positions of the object and image. (b) Use the ray diagram to infer the signs in the equation relating the object and image distances. (c) You want to use a lens with focal length f to look at a beetle with magnification M. How far from the beetle do you need to hold the lens? (d) Check that your answer to part c has units that make sense. (e) Analyze the physical and mathematical behavior of your answer for $M < 1$.

3 An LED lightbulb is positioned between a flat mirror and a converging lens with a focal length of 15 cm. Defining an appropriate coordinate x, the mirror is at $x = 0$, the LED at $x = 10$ cm, and the lens at $x = 20$ cm. Give the x coordinate of the resulting real image.

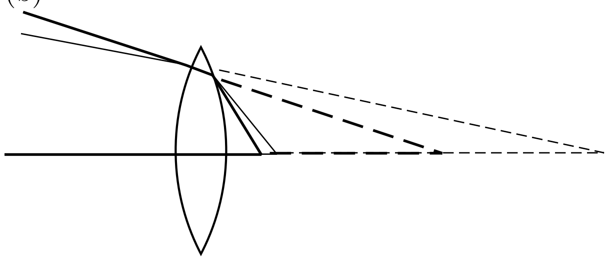
4 (a) Starting from Snell's law, derive an inequality giving the condition for total internal refraction in a material of index of refraction n, when it is surrounded by air or vacuum with index of refraction 1.
(b) In 2011, researchers in Korea created an artificial material that set a new record for the highest index of refraction, 38.6. The material works with radio waves, not visible light. Apply your answer from part a to this material.

Answer to problem 1
All four can be used. The diverging mirror and lens only form virtual images. The converging mirror and lens will form virtual images if the object distance is less than the focal length.

Answer to problem 2
(a)



(b)



Increasing θ_o increases θ_i, so the signs are opposite. Since θ_o is bigger than θ_i, we have $\theta_o - \theta_i = \theta_f$, and $1/d_o - 1/d_i = 1/f$.
(c) Eliminating $d_i = Md_o$ gives $d_o = (1 - 1/M)f$.
(d) Both d_o and f have units of meters, and the factor in parentheses is unitless, so the units make sense.
(e) Setting $M < 1$ results in $d_o < 0$. That doesn't make sense in our sign convention, where all the quantities are positive by definition. Physically, you can't get $M < 1$ in this setup, and that's what the anomalous mathematical result is telling us.

Answer to problem 3
The first images that are formed are both virtual: one made by the mirror and one by the lens. To find a real image, we need to consider secondary images, i.e., images of images. The only secondary image is formed as follows. First the mirror forms a virtual image at $x_1 = -10$ cm. The lens then makes a real image of this virtual at some position x_2, which we will now find.

This is a real image, and a shortcut for finding the signs, as discussed in class, is that because the rays can be time-reversed for a real image, the object and image must be treated symmetrically. That means that the signs must be ++. We then have $1/d_o + 1/d_i = 1/f$. Solving for d_i and plugging in $d_o = 30$ cm gives $d_i = 30$ cm as well. The result is $x_2 = 50$ cm.

Answer to problem 4
(a) Snell's law gives $n \sin\theta_1 = \sin\theta_2$, or $\theta_2 = \sin^{-1}(n\sin\theta_1)$, where 1 is the interior and 2 is the exterior. Total internation reflection occurs when this expression is undefined, which happens when $n\sin\theta_1 > 1$, or $\theta_1 > \sin^{-1}(1/n)$.
(b) Plugging in gives 1.48 degrees. This is a remarkably small angle. Almost any ray inside the material will be completely reflected, the only exception being rays that approach the surface from a direction extremely close to the normal.