Practice Exam 4 for Physics 206

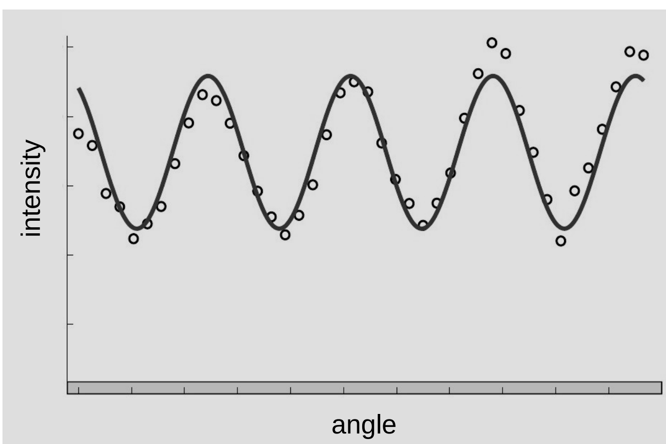
Useful Data

gravitational constant	$G = 6.67 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$
Coulomb constant	$k = 8.99 \times 10^{9}\ \mathrm{J{\cdot}m/C^2}$ or $\mathrm{N{\cdot}m^2/C^2}$
Boltzmann's constant	$k = 1.38 \times 10^{-23}\ \mathrm{J/K}$
quantum of charge	$e = 1.60 \times 10^{-19}\ \mathrm{C}$
speed of light	$c = 3.00 \times 10^{8}\ \mathrm{m/s}$
Planck's constant	$h = 6.63 \times 10^{-34}\ \mathrm{J{\cdot}s}$
absolute zero	$-273.15\,°\mathrm{C}$
speed of sound in air	340 m/s
mass of electron	9.11×10^{-31} kg

1 (a) A sodium discharge tube produces yellow light with a wavelength of 589 nm. Find the energy of a photon of this light.
(b) If this light is diffracted through a grating with 600 lines per millimeter, what is the highest m that will be observable?

2 Various physicists have attempted to diffract larger and larger objects, and it has even been proposed that it might be possible to diffract a virus in the near future. So far, the largest object ever diffracted, by Eibenberger *et al.* in 2013, was a large fluorocarbon molecule containing 810 atoms. The diffraction pattern is shown in the figure (CC-BY license). The circles are the experimental data points. The curve is a fit to the data. The dark strip at the bottom shows the level of background signals.

The molecule used in this experiment had a mass of 1.6810×10^{-23} kg. A beam of these molecules was prepared with a velocity of 85 m/s, and the beam was sent through a diffraction grating with a spacing of 266 nm. Find the angular spacing of the resulting diffraction pattern, in radians.



3 Find the two mistakes in the following attempt to calculate the ground-state energy of a particle in a box in one dimension.

wrong: *A particle of mass m is in a box of length L. The ground state has wavelength $\lambda = L$. The frequency is related to the wavelength by $c = f\lambda$, so $f = c/\lambda = c/L$. The energy is $E = hf = hc/L$.*

4 (a) The energy of states of the hydrogen atom is given by $E = -A/n^2$, where n is an integer. How does this apply to atoms with atomic number greater than 1?
(b) When an atom emits a photon, explain based on the laws of physics what determines the energy of the photon.

Additional practice:
In the following groups of problems, one was assigned randomly to each student.
Ch. 32, problems 4, 6, 9, and 14.
Ch. 35, problems 2 and 9.

Answer to problem 1

(a) The energy of a photon is $E = hf$, and the frequency is $f = c/\lambda$ (because light travels at c). The result is $E = hc/\lambda = 3.37 \times 10^{-19}$ J.

(b) We have $m\lambda = d \sin\theta$, or $\theta = \sin^{-1}(m\lambda/d)$. Since the inverse sine function's range only goes up to 1, the greatest m we can observe will be the greatest one for which $m\lambda/d \leq 1$. To estimate this, we can calculate $m \approx d/\lambda$. The spacing d is (1/600) mm $= 1.67 \times 10^{-}6$ m, so our estimate is $m \approx 2.8$. This tells us that $m = 2$ will exist but $m = 3$ will not.

Answer to problem 2

The wavelength is $\lambda = h/p = h/mv$. The angular spacing of the diffraction pattern, in radians, is $\Delta\theta = \lambda/d = h/mvd = 1.7 \times 10^{-6}$. This is small, but that makes sense. The angle is inversely proportional to m, so if your goal is to measure a diffraction pattern for a large object, you're going to end up having to measure very small angles.

Answer to problem 3

Error 1: The longest wavelength comes when we make a standing wave pattern with one hump. In this wave pattern, *half* a wavelength fits in the box, not a whole wavelength.

Error 2: It's not true that $c = f\lambda$. This particle has mass m, which is presumably not zero, so it doesn't go at the speed of light. The right method would have been to calculate $p = h/\lambda$, then find the kinetic energy using $K = (1/2)mv^2$ (assuming the motion is nonrelativistic).

Answer to problem 4

(a) It doesn't apply at all to atoms with atomic number greater than 1. For these atoms, the set of energy levels is extremely complicated, and the complication grows exponentially with atomic number. There is no useful way to label the states with a single integer n, and there is no simple formula for the energy of the states. Suppose we have an atom with three electrons. Roughly speaking, we can imagine putting *each* of these electrons into one of the standing-wave patterns that is labeled by quantum numbers n, ℓ, and ℓ_z. The state of the whole atom would then be described by a long list of numbers, n_1, n_2, n_3, ℓ_1, ℓ_2, ... Even this is just an approximation, because the electrons repel each other electrically, and this interaction should really be taken into account.

(b) The relevant law of physics is conservation of energy. If the atom changes energy by a certain amount, conservation of energy dictates that this amount of energy go into the photon that is emitted. Therefore the energy of the photon is determined by the *difference* in energy between the two states of the atom.