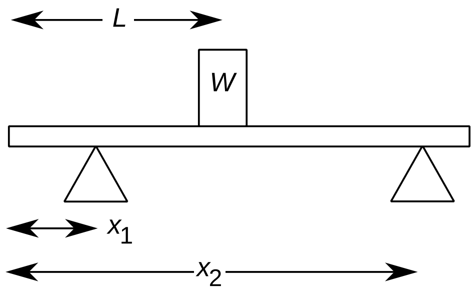
Practice Exam 4 for Physics 221

1 In class we did several demonstrations using a bicycle wheel with a solid rubber tire. Suppose that to get the wheel spinning, we applied a force of 100 N for 0.30 s. Let the radius of the wheel be 25 cm, and its mass 5.0 kg, which we assume to be entirely at the rim. Find the wheel's rotational frequency f.

2 The figures show a massless plank held up by supports 1 and 2. A weight W sits on the plank. Find the force F_1 exerted by support 1.



3 The real estate market in low earth orbit is heating up, and investor Frannie Flipper has decided to buy an old, abandoned space station, fix it up, and sell it. Unfortunately, the station has been hit by three pieces of debris over the years, and may be hit by a fourth soon. The following table gives the momentum vector for each impact, and the radius vector at which it hit the station, relative to the station's center of mass. Throughout this problem, you can ignore units.

	$\mathbf{r}$	$\mathbf{p}$
impact 1	$\hat{\mathbf{x}}+\hat{\mathbf{z}}$	$\hat{\mathbf{y}}$
impact 2	$\hat{\mathbf{z}}$	$\hat{\mathbf{x}}-\hat{\mathbf{y}}$
impact 3	$\hat{\mathbf{x}}$	$\hat{\mathbf{y}}+\hat{\mathbf{z}}$
impact 4		$\hat{\mathbf{y}}$

Aside from the damage done by the impacts, they have also caused the station to start rotating. Frannie decides to take advantage of the fourth impact to cancel out the rotation. By making a microscopic course adjustment to the station, she can control the $\mathbf{r}$ vector at which the fourth piece of junk hits the station. Find an $\mathbf{r}$ vector that will produce the desired effect. (The solution is not unique.)

4 A mechanical oscillator is redesigned so that its mass is greater by 17.3%, without changing its Q. Find the effect that this has on the width of the resonance.

5 Simple harmonic motion seems like a very special and mathematically simplified model of a vibration. Why, then, is the model so often accurate for small-amplitude vibrations?

Answer to problem 1

The force F produces a torque $\tau = rF$, and the definition of torque is $\tau = dL/dt$, which equals $\Delta L/\Delta t$ since the force is assumed to be constant. Since the initial angular momentum is zero, we have $L = rF\Delta t = I\omega$, so $f = rF\Delta t/2\pi I$. The wheel's mass is concentrated at its rim, so in the moment of inertia $I = \int r^2 dm$, the r^2 becomes a constant factor, and $I = r^2 \int dm = r^2 m$. The final result is

$$f = \frac{F\Delta t}{2\pi m r} = 3.8 \text{ Hz}.$$

This seems fairly reasonable compared to reality.

Answer to problem 2

Let positive be up. The total vertical force is zero:

$$F_1 + F_2 - W = 0.$$

Let the axis be on the left, and counterclockwise positive. The total torque is zero:

$$x_1 F_1 + x_2 F_2 - LW = 0.$$

Eliminating F_2 gives

$$F_1 = \frac{L - x_2}{x_1 - x_2} W.$$

The units make sense. It also makes sense that the answer vanishes when $L = x_2$, blows up when $x_1 = x_2$, and is unphysical when $L < x_1$.

Answer to problem 3

The angular momenta delivered by the first three impacts are:

$$\begin{aligned}
\mathbf{L}_1 &= (\hat{\mathbf{x}} + \hat{\mathbf{z}}) \times (\hat{\mathbf{y}}) = -\hat{\mathbf{x}} + \hat{\mathbf{z}} \\
\mathbf{L}_2 &= (\hat{\mathbf{z}}) \times (\hat{\mathbf{x}} - \hat{\mathbf{y}}) = \hat{\mathbf{x}} + \hat{\mathbf{y}} \\
\mathbf{L}_3 &= (\hat{\mathbf{x}}) \times (\mathbf{y} \hat{+} \hat{\mathbf{z}}) = -\hat{\mathbf{y}} + \hat{\mathbf{z}}
\end{aligned}$$

The sum of these is $2\hat{\mathbf{z}}$, so we need an $\mathbf{r}$ vector such that $\mathbf{r} \times \hat{\mathbf{y}} = -2\hat{\mathbf{z}}$. A vector that will work is $-2\hat{\mathbf{x}}$. We could add any y component onto this and it wouldn't matter.

Answer to problem 4

We have $\omega = \sqrt{k/m}$, so the frequency of the resonance is proportional to $m^{-1/2}$. The width of the resonance is proportional to the frequency, for a fixed Q, so the width is also proportional to $m^{-1/2}$. Raising m by a factor of 1.173 causes the width to change by a factor of $1.173^{-1/2} = 0.923$, a decrease of 7.7%.

Answer to problem 5

Our model of simple harmonic motion assumes that there is no friction and that the restoring force is a linear function of the distance from equilibrium. Friction is often small. The assumption of a linear restoring force may seem very special and unrealistic, but any smooth function looks linear under a microscope, so that assumption is essentially guaranteed to be a good one for small enough oscillations.