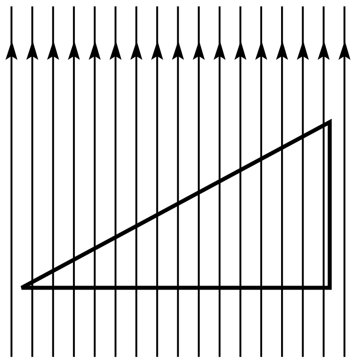
Practice Exam A for Physics 222 — covering Physics 221 and Fields and Circuits ch. 1-3

1 When an electric current is sent through a uniformly would cylindrical coil of wire, the resulting magnetic field is approximately confined to the interior of the cylinder, and is nearly constant within that volume. Coils 1 and 2 are different in size and have different magnetic fields inside. Coil 2's volume is greater by 3%, and its magnetic field is smaller by 1%. Compare the energy stored in coil 2 to the energy stored in coil 1.

2 The figure shows a side view of a uniform electric field E and a surface in the shape of a prism whose cross-section is a right triangle.



The lower leg of the triangle has width w, the lower left angle is θ, and the length of the prism (in and out of the page) is L. Verify Gauss's law by direct calculation.

3 Three identical particles of charge q are fixed at three corners of a square with sides of length b. Find the magnitude of the electric field at the empty corner. Simplify your answer.

4 An unknown particle is subjected to a known electric field, and its acceleration is observed. Explain why this only tells us about the particle's charge to mass ratio q/m, not its charge or its mass individually.

5 Describe the divergences of the earth's gravitational and magnetic fields.

Answer to problem 1

The energy in each solenoid goes like $U \propto B^2 v$, so

$$\begin{aligned}\frac{U_2}{U_1} &= \left(\frac{v_2}{v_1}\right)\left(\frac{B_2}{B_1}\right)^2 \\ &= (1.03)(1/1.01)^2 \\ &= 1.01.\end{aligned}$$

So coil 2's energy will be greater by 1%.

Answer to problem 2

Because the field is uniform there is no charge anywhere in this region. Therefore Gauss's law requires that the total flux through the prism be zero. Because each of the prism's five faces is flat, each has a well defined total area vector. Since field is also uniform, we don't need to do an integral to find the flux. The total flux is $\mathbf{B}_1 \cdot \mathbf{A}_1 + \ldots$, summing over all five faces. The area vector is perpendicular to the surface on each face, so on the front, back, and right faces, it is perpendicular to the field, and each of those dot products is zero, causing those three terms to vanish.

So we only need to consider the fluxes through two sides: the top and the bottom. On the bottom face, the area vector has magnitude wL and points down, so the flux through that face is $\Phi_{\text{bottom}} = -BLw$. The hypotenuse of the triangle is $w/\cos\theta$, so the area of the top face is $Lw/\cos\theta$. The dot product of the field and area vectors on the top face is $\Phi_{\text{top}} = B(\text{area})\cos\theta = B(Lw/\cos\theta)\cos\theta = BLw$. Adding up these two fluxes gives $\Phi_{\text{bottom}} + \Phi_{\text{top}} = 0$, as required by Gauss's law.

Answer to problem 3

Let positive x be to the right and positive y up, and let the square be aligned with the axes. Let the empty corner be the one on the lower left, and put this at the origin O. The contribution to the field at O from the lower right charge has components

$$E_{1x} = -kq/b^2 \qquad E_{1y} = 0.$$

The field from the upper left charge has

$$E_{2x} = 0 \qquad E_{2y} = -kq/b^2.$$

The diagonally opposite charge lies at a distance $\sqrt{2}b$, so the magnitude of its field at O is $E_3 = kq/2b^2$. The x and y components of this field are both $(-\cos 45\,°)E_3$, so

$$E_{3x} = E_{3y} = -kq/2^{3/2}b^2.$$

Adding components gives a total field with components

$$E_x = E_y = -(kq/b^2)(1 + 2^{-3/2}).$$

Its magnitude is $|E| = \sqrt{F_x^2 + F_y^2} = (kq/b^2)(\sqrt{2} + 1/2)$.

Answer to problem 4

The force acting on the charge is $q\mathbf{E}$. Newton's second law then gives $\mathbf{a} = \mathbf{F}/m = (q/m)\mathbf{E}$. This expression only depends on q/m.

Answer to problem 5

Gauss's law for magnetism tells us that the divergence of a magnetic field is always zero, so this is true for the earth's field. Its field lines form closed loops.

Mass is to the gravitational field as charge is to the electric field. The divergence of the gravitational field is the mass density (multiplied by some constants). This tells us that the divergence of the earth's gravitational field is zero outside the earth, and nonzero inside the earth. If the density of the earth is roughly constant (which it sort of is), then the divergence is roughly constant on the interior. Because the density is positive, the divergence is positive.