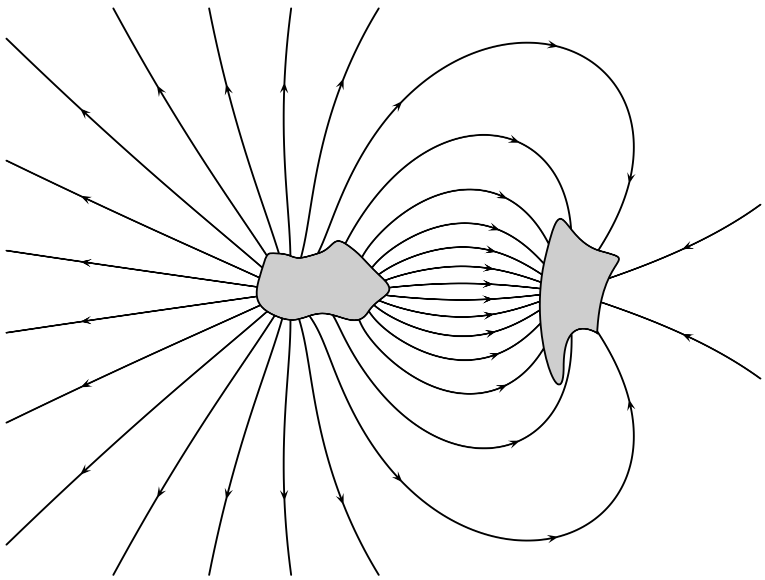
Practice Exam B for Physics 222 — covering Physics 221 and Fields and Circuits ch. 1-6

1 The figure shows a static electric field pattern created by the two gray objects.



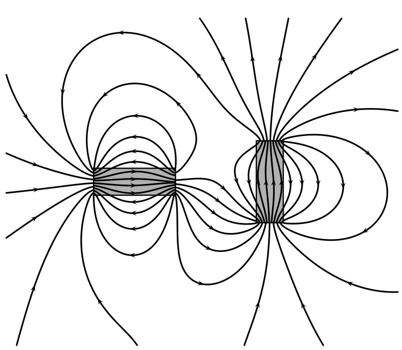
(a) What can you infer about the objects? Could they be conductors? Can you tell anything about their charges?

(b) Suppose that you wanted to replace one of the objects with a copper *sphere*, and still have the same field pattern everywhere except inside the sphere. On the diagram, draw the required size and location of the sphere.

2 A mouse goes running across your kitchen floor with speed u. It's wearing a little backpack that holds a charge q. The magnetic field where you live is horizontal and has magnitude b. If the mouse is free to run in any direction, describe the set of all possible force vectors that could act on it.

3 A certain region of space has an electric potential $\phi = ax^4$, where $a > 0$. Sketch the field pattern, and find an equation for the charge density in terms of a and x.

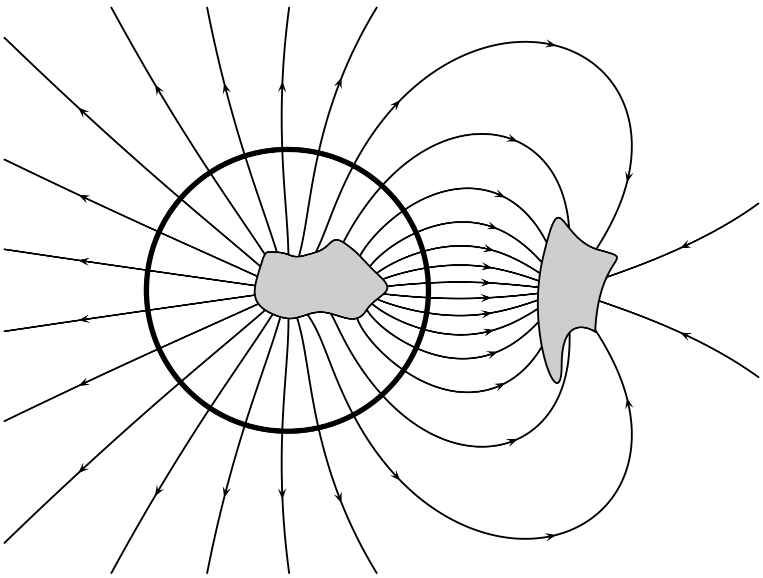
4 The figure shows the fields of two bar magnets. Infer the direction of the torque acting on the right-hand magnet. (Due to limitations in the software used to produce the plot, the magnetic field lines have some barely visible discontinuities at the ends of the magnet. Ignore these, which do not affect the result of the problem.)



5 At a certain point in space the electric and magnetic fields are given by $\mathbf{E} = a\hat{\mathbf{x}} + a\hat{\mathbf{y}}$ and $\mathbf{B} = b\hat{\mathbf{x}} - b\hat{\mathbf{y}}$. (a) What is the direction of energy flow? (b) Can you tell whether these fields could be part of an electromagnetic plane wave?

Answer to problem 1
(a) They can't be conductors, because the field lines aren't perpendicular to their surfaces. The left-hand object has field lines coming out of it, so it must be positively charged, while similar reasoning shows that the right-hand one must be negatively charged. The charges don't look equal in magnitude. There are 28 field lines coming out of the left one, and only 16 going into the right one, so it looks like $|q_{\text{left}}| > |q_{\text{right}}|$. However, we don't expect to be able to make detailed quantitative conclusions (such as $q_{\text{left}}/q_{\text{right}} = -28/16$), because a two-dimensional diagram like this can't really represent all the details of a three-dimensional field pattern.

(b) Copper is a conductor, so its surface will be an equipotential, and field lines will be perpendicular to it. We need to draw a circle somewhere so that it is everywhere perpendicular to the field lines. There is really only one place where this can be done:



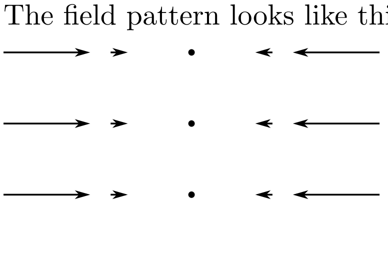
Of course there is a little ambiguity when attempting this by eye, but for example we clearly can't get it to work if we try to cut the diameter in half, or place our circle around the right-hand object.

Answer to problem 2
The force is $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. If nonzero, then it's guaranteed to be perpendicular to both $\mathbf{v}$ and $\mathbf{B}$, which means that it must be vertical. The magnitude can range from zero to $|q|ub$, as the angle between $\mathbf{v}$ and $\mathbf{B}$ varies from parallel to perpendicular. Either an upward or a downward force is possible, based on the right-hand rule. Therefore the set of possible forces is the set of all vectors of the form $f\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is a vertical unit vector and $-|q|ub \le f \le |q|ub$. (If I was the mouse, I would plan things so that the force was either downward, for extra traction, or upward, in order to fly up in the air and escape cats and terriers.)

Answer to problem 3
The electric field is minus the gradient of the potential, which in this effectively one-dimensional problem is the same as minus the derivative,

$$E = -\frac{\mathrm{d}\phi}{\mathrm{d}x} = -4ax^3.$$

The field pattern looks like this:



To find the charge density, we take the divergence of the electric field, which here is just another derivative:

$$\rho = \frac{1}{4\pi k}\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{-3a}{\pi k}x^2.$$

It makes sense that the charge density is negative, since the field lines are disappearing into the center.

Answer to problem 4

The most visually obvious feature of the diagram is that there is a large number of field lines going up and left from the bottom side of the right-hand magnet. There is tension parallel to the field lines, so this will make a strong clockwise torque. Although there are field lines creating tension on the right, there aren't as many of them, so this will be a net clockwise torque.

At the top of the right-hand magnet, we also see an imbalance of tension, this time with more tension on the right. This is again a net clockwise torque.

Adding these two clockwise torques will clearly produce a total torque in the clockwise direction.

The field also produces pressure in the direction perpendicular to the field lines. However, the pressures at the waist of the right-hand magnet look nearly balanced, and in any case these pressures acting near its center will not produce much torque, if any, about the center.

Answer to problem 5

(a) The Poynting vector is in the direction of $\mathbf{E} \times \mathbf{B}$. The two vectors lie in the x-y plane, so their cross product will be in the z direction. The right-hand-rule tells us that it is in the $-z$ direction (assuming that (x, y, z) is a right-handed coordinate system). So for example if x is to the right and y toward the top of the page, then the energy flow is into the page.

(b) If this is to be an electromagnetic plane wave, then the electric and magnetic fields have to be perpendicular to each other, and they also have to be in the ratio $|\mathbf{E}|/|\mathbf{B}| = c$. We can verify from the given information that they are perpendicular, e.g., by computing their dot product. If this is to be an electromagnetic plane wave, then we must also have $a/b = c$.