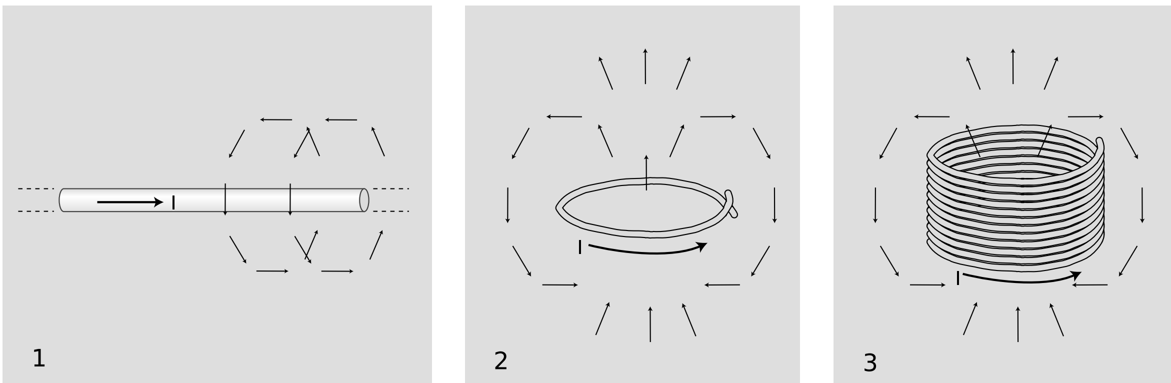
Practice Exam D for Physics 222 — covering Physics 221 and Fields and Circuits ch. 1-11

Useful Data

gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Coulomb constant	$k = 8.99 \times 10^9$ J·m/C^2 or N·m^2/C^2
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
quantum of charge	$e = 1.60 \times 10^{-19}$ C
speed of light	$c = 3.00 \times 10^8$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
absolute zero	$-273.15\,°$C
speed of sound in air	340 m/s
mass of electron	9.11×10^{-31} kg

Some Magnetic Fields



1. Field created by a long, straight wire carrying current I:

$$B = \frac{k}{c^2} \cdot \frac{2I}{r}$$

Here r is the distance from the center of the wire. The field vectors trace circles in planes perpendicular to the wire, going clockwise when viewed from along the direction of the current.

2. Field created by a single circular loop of current:
The field vectors form a dipole-like pattern, coming through the loop and back around on the outside. Each oval path traced out by the field vectors appears clockwise if viewed from along the direction the current is going when it punches through it. There is no simple equation for a field at an arbitrary point in space, but for a point lying *along the central axis* perpendicular to the loop, the field is

$$B = \frac{k}{c^2} \cdot 2\pi I b^2 \left(b^2 + z^2\right)^{-3/2} \qquad ,$$

where b is the radius of the loop and z is the distance of the point from the plane of the loop.

3. Field created by a solenoid (cylindrical coil):
The field pattern is similar to that of a single loop, but for a long solenoid the paths of the field vectors become very straight on the inside of the coil and on the outside immediately next to the coil. For a sufficiently long solenoid, the interior field also becomes very nearly uniform, with a magnitude of

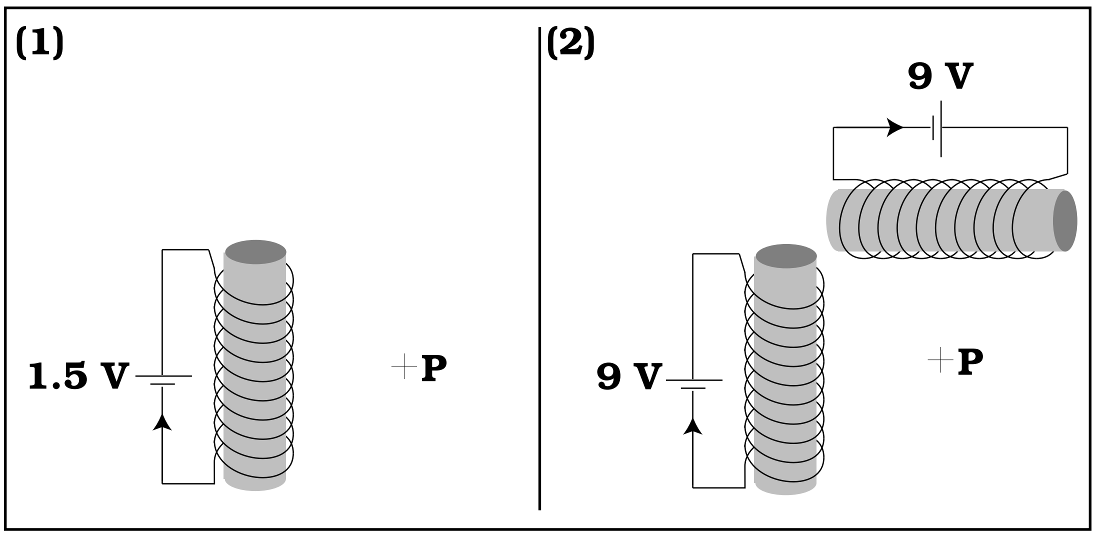
$$B = \frac{k}{c^2} \cdot 4\pi I N/\ell \qquad ,$$

where N is the number of turns of wire and ℓ is the length of the solenoid. The field near the mouths or outside the coil is not constant, and is more difficult to calculate. For a long solenoid, the exterior field is much smaller than the interior field.

1 Let a wire carry current $I > 0$ along the x axis, flowing the positive x direction. A second wire, this one lying along the y axis, carries $2I$ in the negative y direction. (The wires have to make a slight detour to avoid one another, but we assume that this is negligible. Both wires extend infinitely far in both directions.) Suppose that at a certain point (x, y) in the x-y plane, there is zero magnetic field. Relate x to y, simplifying the equation as much as possible.

2 Diagram 1 shows a solenoid made by wrapping wire around a toilet paper roll. It is powered by a 1.5-V battery, and produces a field of 10 mT at point P. Point P is directly across from the center of the solenoid. The arrows show the direction of the current in each circuit.

(a) Indicate with an arrow the direction of the magnetic field at point P in diagram 1. (If you think you might get the direction of the field reversed, just give it your best guess and continue with the problem. You will get nearly full credit if you do everything right except that all your fields are in the opposite direction compared to what they should be.)
(b) An electron is at point P, moving to the right. Find the direction of the force acting on it.
(c) In diagram 2, a second, identical solenoid is added, and 9 V batteries are used to power both solenoids. What is the magnitude of the magnetic field at point P in diagram 2?
(d) Indicate with an arrow the direction of the magnetic field in diagram 2.



Question 2

3 An experiment shows that when a particular magnetic compass's needle is released from rest, in an orientation perpendicular to the earth's field at a particular location, its maximum angular velocity in its later motion is ω_1. (Damping is weak.) Suppose that we wish to start the needle at rest in its equilibrium position, but kick it with some *initial* angular velocity ω_2, such that its motion will carry it all the way around. Find the minimum value of $|\omega_2|$ in terms of ω_1.

4 Consider the electromagnetic field

$$\mathbf{E} = ax\hat{\mathbf{x}}$$
$$\mathbf{B} = bt\hat{\mathbf{y}},$$

where a and b are nonzero constants. Show that this field violates Maxwell's equations.

Answer to problem 1

The first diagram on the front page of the exam shows the direction of the field surrounding a wire, and below it is given an equation that tells us that $B \propto I/r$. To make the fields cancel, we need them to be in opposite directions, which happens in quadrants II and IV. We also need them to be equal in magnitude, which means that we have to be twice as far from the vertical wire as from the horizontal one. Therefore the points where the fields cancel are given by the condition $y = -x/2$.

Answer to problem 2

(a) The current is counterclockwise as seen from above, so the orientation is the same as in the figure on the first page where the formula for the interior field is given. Therefore the field at P is down.

(b) For a velocity pointing to the right and a magnetic field pointing down, the right-hand rule gives a force pointing into the page. Since the electron is negatively charged, the force is out of the page.

(c) Instead of the 1.5 V battery, we now have a 9 V battery driving each coil. Therefore the current in each coil is 6 times greater than before, and each coil's contribution to the magnetic field is 60 mT rather than 10 mT. Referring to the diagram, the second solenoid's contribution to the field is to the left. Let positive x be to the right and positive y up. Then the original solenoid gives $B_{1x} = 0$, $B_{1y} = -60$ mT, and the new solenoid gives $B_{2x} = -60$ mT, $B_{2y} = 0$. The total field has $B_x = -60$ mT and $B_y = -60$ mT, so its magnitude is $\sqrt{B_x^2 + B_y^2} = 84$ mT.

(d) The x and y components are negative and equal to each other, so the field points down and to the left at a 45 degree angle.

Answer to problem 3

The magnetic potential energy of the compass needle is $-\mathbf{m} \cdot \mathbf{B}$. In terms of the magnitudes m and B, this energy equals $-mB$ when the needle is in the equilibrium orientation, 0 when it is perpendicular to the earth's field, and $+mB$ when it is antialigned. For the first experiment, conservation of energy gives

$$0 + 0 = -mB + K_1,$$

while in the second, the motion has to extend through the antialiged position, so

$$-mB + K_2 \geq mB + 0.$$

We're interested in the *minimum* value of $|\omega_2|$, which is the case where the inequality becomes an equality. Then $K_2 = 2K_1$. Since $K \propto \omega^2$, this means $|\omega_2| = \sqrt{2}|\omega_1|$.

Answer to problem 4

One of Maxwell's equations is

$$\operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

For the given fields, the right-hand side is nonzero. However, we can easily tell by visualizing a curl-meter that the left-hand side is zero.