Practice Exam E for Physics 222 — covering Physics 221 and Fields and Circuits ch. 1-15

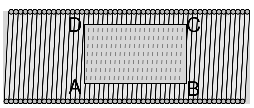
1 Compute the following.
(a) $|i(1+i)(-1-i)(-2i)|$
(b) $\arg[i(1+i)(-1-i)(-2i)]$
(c) $\ln(-1)$

2 (a) We want to get an AC current with amplitude I to flow through an inductance L at frequency f. What voltage must we apply?
(b) Show that your answer has units that make sense.
(c) Evaluate your result numerically for $I = 120$ mA, $L = 5.0$ mH, and $f = 1700$ kHz.

3 (a) Use the definition of inductance $U = (1/2)LI^2$ to express the henry in terms of meters, kilograms, seconds, and coulombs.
(b) The Q of an LRC circuit equals $R^{-1}L^{1/2}C^{-1/2}$. Verify that this expression is unitless.

4 Jane is stranded aboard an abandoned alien space station and is scrounging around for stuff she can use to build a radio and call for help. She is testing a squishy orange component that smells like tacos. When she applies an AC voltage to it at a certain frequency, it responds with a current that leads the voltage by 87° in phase. What would be her natural interpretation of this observation, and how could she further test and refine it?

5 Fred decides to apply Ampère's law to a solenoid using the Ampèrian surface shown below, a rectangle of width w and height h whose midline is on the axis.



Fred reasons that by symmetry, the contribution to $\int \mathbf{B} \cdot d\boldsymbol{\ell}$ is the same on sides AB and CD, while it's zero on BC and DA, so the circulation around the whole rectangle is $\int \mathbf{B} \cdot d\boldsymbol{\ell} = 2wB$. He then gets

$$B = \frac{8\pi k N I}{c^2 \ell}.$$

Fred's result is wrong. How did he mess up?

Answer to problem 1
(a) The magnitude of the product is the product of the magnitudes of the factors, $(1)(\sqrt{2})(\sqrt{2})(2) = 4$.
(b) The argument of the product is the sum of the arguments, $\pi/2 + \pi/4 + 5\pi/4 + 3\pi/2 = 7\pi/2$, which is equivalent to $-\pi/2$.
(c) We want a complex number z such that $e^z = -1$. Euler's formula gives $e^{\phi i} = \cos\phi + i\sin\phi$, so we want $\phi = \pi$. Therefore $\ln(-1) = \pi i$.

Answer to problem 2
(a) The impedance is $i\omega L$, so $V = iI\omega L = 2\pi i I f L$.
(b) The equation $Z = i\omega L$ tells us that a henry can be expressed as 1 $\Omega\cdot$s. The units of our result are then $\text{A}\cdot\text{s}^{-1}\cdot\Omega\cdot\text{s} = \text{A}\cdot\Omega = \text{V}$.
(c) Converting to the base SI units and plugging in, we get $|V| = 6.4 \times 10^3$ V. This is a lot of voltage, but that sort of makes sense, because we're trying to drive a pretty big inductance at a high frequency, and inductances don't like high frequencies. That's why they're called "chokes."

Answer to problem 3
(a)

$$\text{H} = \frac{\text{J}}{\text{A}^2} = \frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{C}^2/\text{s}^2} = \frac{\text{kg}\cdot\text{m}^2}{\text{C}^2}$$

(b) The obvious brute-force method would be to break everything down into the basic units, as in part a. A more efficient approach is to use the equations for impedance, $Z_C = -i/\omega C$ and $Z_L = i\omega L$, to express a henry as 1 $\Omega\cdot$s and a farad as 1 s/Ω. We then have units of

$$(\Omega)^{-1}(\Omega\cdot\text{s})^{1/2}(\text{s}/\Omega)^{-1/2},$$

which does work out to be unitless.

Answer to problem 4
We usually describe these things in terms of the phase angle of the impedance, which is the phase of the voltage relative to the current rather than that of the current relative to the voltage. In these terms, she has observed that the voltage lags behind the current by this angle, i.e., that the impedance has a phase of $-87\,°$. The orange thingie is behaving like an almost purely capacitive impedance ($Z = -i/\omega C$). If it's really an alien capacitor, she could change the frequency of the voltage and see whether the current is proportional to the frequency ($I = V/Z \propto \omega$). The fact that the phase angle is not exactly $-90\,°$ could also be investigated further. It could for example be because there is a small resistance in series with the capacitance (perhaps just the resistance of the wires), or a large resistance in parallel with it. This kind of thing could also be tested by testing the frequency dependence. E.g., if it's a large parallel resistance, then some current should flow even at zero frequency (DC).

Answer to problem 5
Fred made two mistakes. First, the right-hand side of Ampère's law is supposed to involve the current passing through the Ampèrian surface, but Fred's Ampèrian surface doesn't enclose any currents, so the right-hand side should be zero.

Also, the integral defining the circulation depends on the choice of orientation of the surface. For example, let's say that the field is to the right, and Fred chooses the surface to have a counterclockwise orientation. Then the circulation along the top edge is $-wB$, and that along the bottom is wB. They cancel. He could have chosen the other orientation, but there would have again been a cancellation.

So Ampère's law works just fine when applied to Fred's surface, but it tells us $0 = 0$, which we already knew. Or we could say that it verifies that it is possible, as implicitly assumed, for the interior field to be uniform.