Practice Exam 1 for Physics 223 — Simple Nature ch. 5 and sec. 6.1

Useful Data

gravitational constant	$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$
Coulomb constant	$k = 8.99 \times 10^9\ \text{J}\cdot\text{m}/\text{C}^2$ or $\text{N}\cdot\text{m}^2/\text{C}^2$
Boltzmann's constant	$k = 1.38 \times 10^{-23}\ \text{J/K}$
quantum of charge	$e = 1.60 \times 10^{-19}\ \text{C}$
speed of light	$c = 3.00 \times 10^8\ \text{m/s}$
Planck's constant	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$
absolute zero	$-273.15\,^\circ\text{C}$
speed of sound in air	340 m/s
mass of electron	9.11×10^{-31} kg

1 Some animals, such as bats and dolphins, use sonar to sense their environment. In experiments, dolphins have been shown to be able to distinguish small objects such as coins of different sizes. In order to have this kind of centimeter-scale resolution, their sonar must have a wavelength of less than about 1 cm. The speed of sound in water is 1500 m/s. Estimate the frequency of the sonar. Is this an upper bound, or a lower bound?
As a check on your answer, you should find that the frequency is greater than 100 kHz. That frequency represents the upper range of the frequencies that a killer whale can hear. Killer whales eat dolphins, so dolphins have the evolutionary advantage of being able to use their sonar without being heard by their predators.

2 (a) Atmospheric pressure at sea level is 101 kPa. The deepest spot in the world's oceans is a valley called the Challenger Deep, in the Marianas Trench, with a depth of 11.0 km. Find the pressure at this depth, in units of atmospheres. Although water under this amount of pressure does compress by a few percent, assume for the purposes of this problem that it is incompressible.
(b) Suppose that an air bubble is formed at this depth and then rises to the surface. Estimate the change in its volume and radius.

3 Object A is a brick. Object B is half of a similar brick. If A is heated, we have $\Delta S = Q/T$. Show that if this equation is valid for A, then it is also valid for B.

4 Our sun is powered by nuclear fusion reactions, and as a first step in these reactions, one proton must approach another proton to within a short enough range r. This is difficult to achieve, because the protons have electric charge $+e$ and therefore repel one another electrically. (It's a good thing that it's so difficult, because otherwise the sun would use up all of its fuel very rapidly and explode.) To make fusion possible, the protons must be moving fast enough to come within the required range. Even at the high temperatures present in the core of our sun, almost none of the protons are moving fast enough.

(a) For comparison, the early universe, soon after the Big Bang, had extremely high temperatures. Estimate the temperature T that would have been required so that protons with average energies could fuse. State your result in terms of r, the mass m of the proton, and universal constants.
(b) Show that the units of your answer to part a make sense.
(c) Evaluate your result from part a numerically, using $r = 10^{-15}$ m and $m = 1.7 \times 10^{-27}$ kg. As a check, you should find that this is much hotter than the sun's core temperature of $\sim 10^7$ K.

5 Typically the atmosphere gets colder with increasing altitude. However, sometimes there is an *inversion layer*, in which this trend is reversed, e.g., because a less dense mass of warm air moves into a certain area, and rises above the denser colder air that was already present. Suppose that this causes the pressure P as a function of height y to be given by a function of the form $P = P_o e^{-ky}(1 + by)$, where constant temperature would give $b = 0$ and an inversion layer would give $b > 0$. (a) Infer the units of the constants P_o, k, and b. (b) Find the density of the air as a function of y, of the constants, and of the acceleration of gravity g. (c) Check that the units of your answer to part b make sense.

Answer to problem 1
We have $v = f\lambda$ and $\lambda \lesssim 1$ cm, so $f = v/\lambda \gtrsim 150$ kHz. Division flips the direction of the inequality, so this is a lower bound ($\gtrsim$). This is greater than 100 kHz, which is consistent with the given information about killer whales.

Answer to problem 2
(a) We have

$$\begin{aligned} \mathrm{d}P &= \rho g \mathrm{d}y \\ \Delta P &= \int \rho g \mathrm{d}y, \end{aligned}$$

and since we're taking water to be incompressible, and g doesn't change very much over 11 km of height, we can treat ρ and g as constants and take them outside the integral.

$$\begin{aligned} \Delta P &= \rho g \Delta y \\ &= (1.0\ \mathrm{g/cm^3})(9.8\ \mathrm{m/s^2})(11.0\ \mathrm{km}) \\ &= (1.0 \times 10^3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(1.10 \times 10^4\ \mathrm{m}) \\ &= 1.0 \times 10^8\ \mathrm{Pa} \\ &= 1.0 \times 10^3\ \mathrm{atm}. \end{aligned}$$

The precision of the result is limited to a few percent, due to the compressibility of the water, so we have at most two significant figures. If the change in pressure were exactly a thousand atmospheres, then the pressure at the bottom would be 1001 atmospheres; however, this distinction is not relevant at the level of approximation we're attempting here.
(b) Since the air in the bubble is in thermal contact with the water, it's reasonable to assume that it keeps the same temperature the whole time. The ideal gas law is $PV = nkT$, and rewriting this as a proportionality gives

$$V \propto P^{-1},$$

or

$$\frac{V_f}{V_i} = \left(\frac{P_f}{P_i}\right)^{-1} \approx 10^3.$$

Since the volume is proportional to the cube of the linear dimensions, the growth in radius is about a factor of 10.

Answer to problem 3
If the full-sized brick A undergoes some process, such as heating it with a blowtorch, then we want to be able to apply the equation $\Delta S = Q/T$ to either the whole brick or half of it, which would be identical to B. When we redefine the boundary of the system to contain only half of the brick, the quantities ΔS and Q are each half as big, because entropy and energy are additive quantities. T, meanwhile, stays the same, because temperature isn't additive — two cups of coffee aren't twice as hot as one. These changes to the variables leave the equation consistent, since each side has been divided by 2.

Answer to problem 4
(a) Roughly speaking, the thermal energy is $\sim k_B T$ (where k_B is the Boltzmann constant), and we need this to be on the same order of magnitude as ke^2/r (where k is the Coulomb constant). For this type of rough estimate it's not especially crucial to get all the factors of two right, but let's do so anyway. Each proton's average kinetic energy due to motion along a particular axis is $(1/2)k_B T$. If two protons are colliding along a certain line in the center-of-mass frame, then their average combined kinetic energy due to motion along that axis is $2(1/2)k_B T = k_B T$. So in fact the factors of 2 cancel. We have $T = ke^2/k_B r$.
(b) The units are $\mathrm{K} = (\mathrm{J{\cdot}m/C^2})(\mathrm{C^2})/((\mathrm{J/K}){\cdot}\mathrm{m})$, which does work out.
(c) The numerical result is $\sim 10^{10}$ K, which as suggested is much higher than the temperature at the core of the sun.

Answer to problem 5
(a) If the expression $1 + by$ is to make sense, then by has to be unitless, so b has units of m^{-1}. The input

to the exponential function also has to be unitless, so k also has of m^{-1}. The only factor with units on the right-hand side is P_o, so P_o must have units of pressure, or Pa.
(b)

$$\begin{aligned}
\mathrm{d}P &= \rho \mathbf{g} \cdot \mathrm{d}\mathbf{y} \\
&= -\rho g \mathrm{d}y \\
\rho &= -\frac{1}{g}\frac{\mathrm{d}P}{\mathrm{d}y} \\
&= \frac{P_o}{g} e^{-ky}(k + kby - b)
\end{aligned}$$

(c) The three terms inside the parentheses on the right all have units of m^{-1}, so it makes sense to add them, and the factor in parentheses has those units. The units of the result from b then look like

$$\begin{aligned}
\frac{\mathrm{kg}}{\mathrm{m}^3} &= \frac{\mathrm{Pa}}{\mathrm{m/s}^2}\mathrm{m}^{-1} \\
&= \frac{\mathrm{N/m}^2}{\mathrm{m}^2/\mathrm{s}^2} \\
&= \frac{\mathrm{kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{s}^{-2}}{\mathrm{m}^2/\mathrm{s}^2},
\end{aligned}$$

which checks out.