Practice Exam 2 for Physics 223 — through Simple Nature sec. 12.4

Useful Data

gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Coulomb constant	$k = 8.99 \times 10^9$ J·m/C^2 or N·m^2/C^2
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
quantum of charge	$e = 1.60 \times 10^{-19}$ C
speed of light	$c = 3.00 \times 10^8$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
absolute zero	$-273.15\,°$C
speed of sound in air	340 m/s

1 A clarinet can be analyzed as an asymmetric air column, i.e., one in which the reflection is inverting at one end and noninverting at the other. For simplicity, let's assume that all the keys are held down, so that the length of the air column is the full length ℓ of the clarinet.
(a) Draw the wave patterns for the first three modes of vibration.
(b) Find the wavelengths of the three lowest notes, in terms of ℓ.
(c) Find the corresponding frequencies in terms of ℓ and the speed of sound v.
(d) Evaluate these frequencies numerically for $\ell = 60.0$ cm and $v = 340$ m/s.

2 The air column inside your vocal tract (throat and mouth) has resonances which, for the purposes of this problem, we will treat like those of a simple cylindrical tube. If you inhale the helium from a balloon and then say something, your voice sounds very high, like Mickey Mouse. (If you try this, don't do it many times in a row, because you could pass out.)

(a) Explain why this effect occurs, and how it relates to the speed of sound in helium as opposed to air. Based on this observation, is the speed of sound greater or smaller in helium compared to air?

(b) In submarines, the oxygen-nitrogen mix of natural air may be replaced with a mixture of oxygen and helium. (This is in order to reduce the chances of getting the "bends" during rapid dives or ascents.) The effect on the pitch of people's voices is similar to, but not as big as, the effect of breathing pure helium. Suppose two crew members on a submarine have brought along a trumpet and a guitar, and want to play a duet. The sub is near the surface of the water, so the air pressure is not any different from normal atmospheric pressure — only the mixture of gases is unusual. What will be the effect on each instrument? Will they still be in tune with each other? Explain.

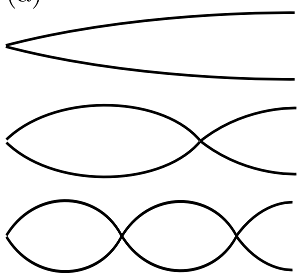
3 An LED lightbulb is positioned between a flat mirror and a converging lens with a focal length of 15 cm. Defining an appropriate coordinate x, the mirror is at $x = 0$, the LED at $x = 10$ cm, and the lens at $x = 20$ cm. Give the x coordinate of the resulting real image.

4 (a) Starting from Snell's law, derive an inequality giving the condition for total internal refraction in a material of index of refraction n, when it is surrounded by air or vacuum with index of refraction 1.
(b) In 2011, researchers in Korea created an artificial material that set a new record for the highest index of refraction, 38.6. The material works with radio waves, not visible light. Apply your answer from part a to this material.

5 A converging lens is used to produce a virtual image. (a) Draw a ray diagram and indicate the positions of the object and image. (b) Use the ray diagram to infer the signs in the equation relating the object and image distances. (c) You want to use a lens with focal length f to look at a beetle with magnification M. How far from the beetle do you need to hold the lens? (d) Check that your answer to part c has units that make sense. (e) Analyze the physical and mathematical behavior of your answer for $M < 1$.

Answer to problem 1

(a)



(b) Based on the drawings, the wavelengths are 4ℓ, $4\ell/3$, and $4\ell/5$.

(c) Using $v = f\lambda$, we find frequencies $v/4\ell$, $3v/4\ell$, and $5v/4\ell$.

(d) The results are 142 Hz, 425 Hz, and 708 Hz.

Answer to problem 2

(a) In the model we're assuming, where the vocal tract is treated as a cylindrical tube, the wavelength is fixed by the length of the tube. (It will be either twice the length of the tube or four times its length, depending on whether the ends are symmetric or asymmetric.) For a fixed wavelength, we have $f = v/\lambda$, so a higher v gives a higher f. The observation that the frequency is higher for helium implies that the speed of sound in helium is greater.

(b) The trumpet is physically analogous to what we discussed in part a. The guitar is different; its pitch is fixed by the speed of waves on the guitar string, which has nothing to do with the surrounding air. The trumpet will go up in pitch, but the guitar will be normal, so the instruments will be out of tune with each other.

Answer to problem 3

The first images that are formed are both virtual: one made by the mirror and one by the lens. To find a real image, we need to consider secondary images, i.e., images of images. The only secondary image is formed as follows. First the mirror forms a virtual image at $x_1 = -10$ cm. The lens then makes a real image of this virtual at some position x_2, which we will now find.

This is a real image, and a shortcut for finding the signs, as discussed in class, is that because the rays can be time-reversed for a real image, the object and image must be treated symmetrically. That means that the signs must be ++. We then have $1/d_o + 1/d_i = 1/f$. Solving for d_i and plugging in $d_o = 30$ cm gives $d_i = 30$ cm as well. The result is $x_2 = 50$ cm.

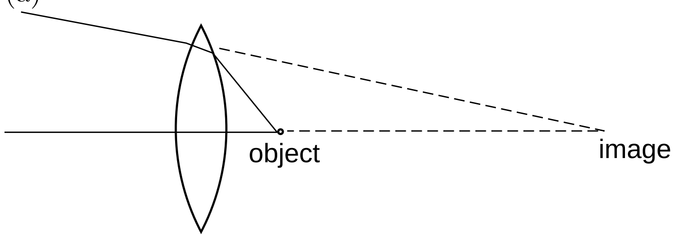
Answer to problem 4

(a) Snell's law gives $n \sin\theta_1 = \sin\theta_2$, or $\theta_2 = \sin^{-1}(n \sin\theta_1)$, where 1 is the interior and 2 is the exterior. Total internation reflection occurs when this expression is undefined, which happens when $n \sin\theta_1 > 1$, or $\theta_1 > \sin^{-1}(1/n)$.

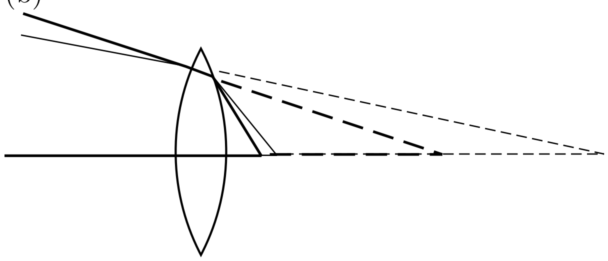
(b) Plugging in gives 1.48 degrees. This is a remarkably small angle. Almost any ray inside the material will be completely reflected, the only exception being rays that approach the surface from a direction extremely close to the normal.

Answer to problem 5

(a)



(b)



Increasing θ_o increases θ_i, so the signs are opposite. Since θ_o is bigger than θ_i, we have $\theta_o - \theta_i = \theta_f$, and $1/d_o - 1/d_i = 1/f$.

(c) Eliminating $d_i = Md_o$ gives $d_o = (1 - 1/M)f$.

(d) Both d_o and f have units of meters, and the factor in parentheses is unitless, so the units make sense.

(e) Setting $M < 1$ results in $d_o < 0$. That doesn't make sense in our sign convention, where all the quantities are positive by definition. Physically, you can't get $M < 1$ in this setup, and that's what the anomalous mathematical result is telling us.