Practice Exam 3 for Physics 223 — up to Simple Nature, sec. 11.6

Useful Data

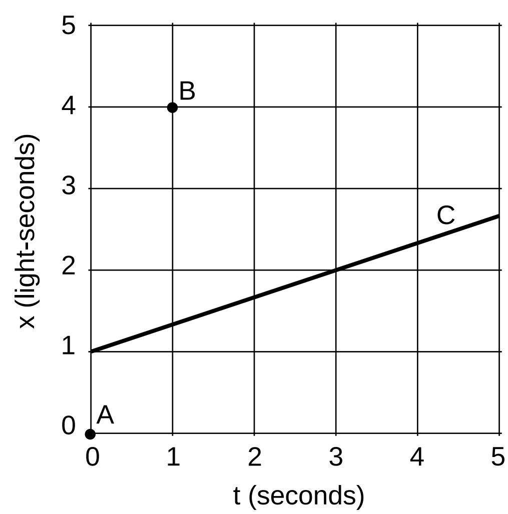
gravitational constant	$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$
Coulomb constant	$k = 8.99 \times 10^9\ \text{J}\cdot\text{m/C}^2$ or $\text{N}\cdot\text{m}^2/\text{C}^2$
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
quantum of charge	$e = 1.60 \times 10^{-19}$ C
speed of light	$c = 3.00 \times 10^8$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$
absolute zero	$-273.15\,°\text{C}$
speed of sound in air	340 m/s

1 (a) Human vision is most sensitive to light with a wavelength of about 550 nm, which is a yellowish color typical of sunlight. Find the frequency of light that has this wavelength.
(b) Suppose that we want to take a certain light wave and make its energy 17 times greater. By what factor do we have to increase its electric field? Its magnetic field?

2 Microwaves with a wavelength of 2.00 cm propagate in the positive x direction, in the space between two metal sheets that are each parallel to the $x - z$ plane. The distance between the sheets is 13.6 cm, and they end at edges lying at $x = 0$. The microwaves reach the end of the sheets and emerge into open space, undergoing single-slit diffraction as they emerge. Find the number of complete diffraction fringes that are observable in the forward (positive x) direction.

3 Our standard definition of the magnetic field **B** includes an element of arbitrariness. Historically, people could instead have defined "the magnetic field" to be $\mathbf{C} = -\mathbf{B}$, i.e., the direction of the field could simply have been reversed. (a) Write down Maxwell's equations in a vacuum in the standard form, then substitute $-\mathbf{C}$ for **B** everywhere, and find the form of the resulting version of Maxwell's equations. (b) Describe the direction of propagation of an electromagnetic wave in terms of the **E** and **C** vectors.

4 The figure shows a graph of position versus time, depicted in some inertial frame of reference. The units on the x axis are light-seconds; a light-second is the distance traveled by light in one second. A and B are events, and C is a graph of the motion of an observer named Chuck.
(a) Find Chuck's velocity in this frame of reference.
(b) What is Chuck's velocity in his own frame of reference?
(c) Is Chuck accelerating, or is he moving inertially, like an object described by Newton's first law? Explain how you know.
(d) A radio beep is emitted from B. Find the time when Chuck receives it.
(e) In some other frame of reference, events A and B are simultaneous. Find the velocity of that frame of reference relative to this one.



Question 4

5 Let R be defined an object's kinetic energy divided by its mass. (a) In nonrelativistic physics, find the object's velocity v in terms of R.
(b) Find the corresponding relativistic equation.
(c) Show that your result from part b makes sense for $R = 0$ and $R \to \infty$.

Answer to problem 1

(a)

$$f = \frac{v}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{5.50 \times 10^{-7}\ \text{m}} = 5.5 \times 10^{14}\ \text{Hz}$$

(b) The electric and magnetic fields in an electromagnetic wave are in fixed proportion, so whatever we do to one of the fields, we have to do the same to the other. The energy density of each field is proportional to the square of the field, so each field has to be increased by a factor of $\sqrt{17} = 4.1$.

Answer to problem 2

Because this is single-slit diffraction, we have a double-width central fringe. A normal, single-width central fringe would have gone from the minimum at $m = -1/2$ to the minimum at $m = 1/2$, but this double-width one goes from $m = -1$ to $m = 1$. After that, we will have single-width maxima running from 1 to 2, -1 to -2, and so on.

Now we need to find the maximum m that exists. The forward direction lies at angles $-90° < \theta < 90°$, so that $\sin\theta$ runs from -1 to 1. We have $m\lambda = d \sin\theta$, so this means that $|m| < d/\lambda = 6.8$. Therefore the last complete fringe on the positive side is the one from $m = 5$ to $m = 6$. That makes 5 single-width fringes on the positive side. Adding in the 5 corresponding fringes on the left, plus the central-width one, we have a total of 11 complete fringes.

Answer to problem 3

(a) In a vacuum, we have $q = 0$ and $I = 0$. The standard form of Maxwell's equations then looks like this: For a closed surface,

$$\Phi_E = 0 \qquad \text{and}$$
$$\Phi_B = 0.$$

For any surface that is not closed, the circulations around the edges of the surface are given by

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{\partial \Phi_B}{\partial t} \qquad \text{and}$$
$$c^2 \oint \mathbf{B} \cdot d\mathbf{s} = \frac{\partial \Phi_E}{\partial t}.$$

On the left-hand sides, substituting $-\mathbf{C}$ for $\mathbf{B}$ simply flips the signs of the magnetic flux and circulation, but the sign flip in the flux through a closed surface doesn't matter, because it's zero anyway. On the right, it also flips the sign of the magnetic flux. The result is:

$$\Phi_E = 0$$
$$\Phi_C = 0$$
$$\oint \mathbf{E} \cdot d\mathbf{s} = \frac{\partial \Phi_C}{\partial t}$$
$$c^2 \oint \mathbf{C} \cdot d\mathbf{s} = -\frac{\partial \Phi_E}{\partial t}.$$

The result is that the equations for the fluxes through a closed surface are unchanged in form, while the only change in the form of the equations for the circulations is that the minus sign is moved out of the third equation and into the fourth.

(b) In the standard formulation, the wave propagates in the direction of $\mathbf{E} \times \mathbf{B}$, i.e., there is a right-hand rule if we state $\mathbf{E}$ first and then $\mathbf{B}$. In terms of the new formulation, the wave would propagate in the direction of $\mathbf{E} \times (-\mathbf{C})$, or $-\mathbf{E} \times \mathbf{C}$. That is, we would have a left-hand rule if $\mathbf{E}$ was stated first and then $\mathbf{C}$.

Answer to problem 4
(a) Chuck moves 1 light-second in 3 seconds, so his velocity is 1/3 in natural units, i.e., $c/3$.
(b) Chuck's velocity is zero in his own frame of reference.
(c) He's moving inertially. We can tell this because his graph is a line.
(d) The beep moves at c, which is depicted on this graph as a line with a slope of ± 1. The part of the expanding wavefront with slope -1 is the one that will reach Chuck. Chuck receives it at $t = 3$ seconds, which is the point where the beep's graph intersects Chuck's.
(e) When we do a Lorentz transformation, the x and t axes rotate like a pair of scissors towards the 45-degree diagonal. To make A and B simultaneous, we have to rotate the x axis enough to make it run through B, which means that as depicted on this graph, it would go 1 unit over and 4 units up. The matching behavior for the t axis would be that it would go 4 units over and 1 up. This is a slope of 1/4, so the velocity of that frame of reference would have to be 1/4, i.e., $c/4$.

Answer to problem 5
(a) $R = KE/m = (mv^2/2)/m = v^2/2$, so $v = \sqrt{2R}$.
(b) $R = m(\gamma - 1)/m = \gamma - 1$. Solving for v gives $v = \sqrt{1 - (R+1)^{-2}}$.
(c) For $R = 0$, we get $v = 0$, which makes sense. For $R \to \infty$, the quantity $(R+1)^{-2}$ approaches zero, so $v \to 1$, i.e., the velocity approaches the speed of light, which also makes sense.