Practice Exam 4 for Physics 223 — up to Simple Nature, sec. 13.2

Useful Data

gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Coulomb constant	$k = 8.99 \times 10^9$ J·m/C^2 or N·m^2/C^2
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
quantum of charge	$e = 1.60 \times 10^{-19}$ C
speed of light	$c = 3.00 \times 10^8$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
absolute zero	$-273.15\,°$C
speed of sound in air	340 m/s
mass of electron	9.11×10^{-31} kg

1 A number in scientific notation has the form $x \times 10^n$, where n is an integer, and x, called the mantissa, is a number between 1 and 10. If we do a long and complicated calculation, then there are arguments suggesting that any given number occurring during the calculation will have a mantissa whose probability distribution is $D(x) = k/x$ (for $1 \le x < 10$), where k is a constant.
(a) Find the constant k.
(b) Find the average value of the mantissa.

2 (a) Suppose that Kim Jong-Un, preparing for a stealthy terrorist attack on US territory, arranges to have radioactive stuff placed inside a shipping container full of Chinese consumer goods, which is being shipped to Long Beach. Consider the cases where the stuff emits alphas, betas, or gammas. Discuss what would need to be done to detect the container at the port in each of these cases.
(b) A hydrogen-2 nucleus can combine with another hydrogen-2 nucleus to create a helium nucleus, with a release of energy. Seawater naturally contains some hydrogen-2 (about 0.0115% of the hydrogen in the H_2O molecules). Should this reaction, then, be expected to occur naturally in the oceans, albeit perhaps at a very low rate because of the small abundance of hydrogen-2?

3 A sodium discharge tube produces yellow light with a wavelength of 589 nm. Find the energy of a photon of this light.

4 A sample of radioactive material is initially a pure sample of a single isotope. This isotope decays to a stable daughter isotope. After time t, the number of atoms of the parent isotope is x, while the number of atoms of the daughter isotope if y. Find the half-life.

Answer to problem 1
(a) Normalization requires that $1 = \int_1^{10} k dx/x = k \ln 10$, so $k = 1/\ln 10$.
(b) The average is $\int_1^{10} xD(x)dx = 9/\ln 10 \approx 3.9$. It makes sense that the average is less than 5.5, which is the midpoint of the interval.

Answer to problem 2
(a) Gammas are highly penetrating, so if it emitted gammas, an inspector at the port could probably detect them from outside the container, using a sufficiently sensitive detector and waiting for long enough to get good statistics. Alphas and betas, however, would be stopped by whatever box the radioactive stuff was inside, so the only way to detect those types of radioactivity would be to empty out the entire container on the dock and comb through it item by item.
(b) The nuclei repel each other electrically, so they can't fuse unless they are initially moving toward one another at very high speeds. Those speeds may occur in the core of the sun or in a nuclear bomb, but they don't exist at the temperatures found in the earth's oceans.

Answer to problem 3
The energy of a photon is $E = hf$, and the frequency is $f = c/\lambda$ (because light travels at c). The result is $E = hc/\lambda = 3.37 \times 10^{-19}$ J.

Answer to problem 4
The probability of survival is measured to be $x/(x + y)$, and should equal $(1/2)^{t/t_{1/2}}$. We then have

$$\begin{aligned} 1 + \frac{y}{x} &= 2^{t/t_{1/2}} \\ t_{1/2} &= \frac{t \ln 2}{\ln(1 + y/x)}. \end{aligned}$$

As a check, let's look at the behavior of the result in the cases where y/x is very large or very small. If y/x is very large, then nearly all the atoms have decayed, so the half-life must have been very short compared to t. This works out mathematically, since we'd be taking the log of a large number in the denominator, which would make the result small. In the case where y/x is very small, the log in the denominator is approximately $\ln 1 = 0$, which would make $t_{1/2}$ large, and that also makes sense physically.