Page 838: The area under the curve from 130 to 135 cm is about 3/4 of a rectangle. The area from 135 to 140 cm is about 1.5 rectangles. The number of people in the second range is about twice as much. We could have converted these to actual probabilities (1 rectangle = 5 cm $\times$ 0.005 cm^{-1} = 0.025), but that would have been pointless because we were just going to compare the two areas.

Page 843: On the left-hand side, $\mathrm{d}N$ is a unitless count, and $\mathrm{d}t$ is an infinitesimal amount of time, with units of seconds, so the units are s^{-1} as claimed. On the right, both $N(0)$ and the exponential factor are unitless, so the only units come from the factor of $1/\tau$, which again has units of s^{-1}.

Page 852: The axes of the graph are frequency and photon energy, so its slope is Planck's constant. It doesn't matter if you graph $e\Delta V$ rather than $W + e\Delta V$, because that only changes the y-intercept, not the slope.

Page 865: Wavelength is inversely proportional to momentum, so to produce a large wavelength we would need to use electrons with very small momenta and energies. (In practical terms, this isn't very easy to do, since ripping an electron out of an object is a violent process, and it's not so easy to calm the electrons down afterward.)

Page 874: Under the ordinary circumstances of life, the accuracy with which we can measure position and momentum of an object doesn't result in a value of $\Delta p \Delta x$ that is anywhere near the tiny order of magnitude of Planck's constant. We run up against the ordinary limitations on the accuracy of our measuring techniques long before the uncertainty principle becomes an issue.

Page 878: No. The equation $K = p^2/2m$ is nonrelativistic, so it can't be applied to an electron moving at relativistic speeds. Photons always move at relativistic speeds, so it can't be applied to them either.

Page 880: Dividing by Planck's constant, a small number, gives a large negative result inside the exponential, so the probability will be very small.

Page 901: The original argument was that a kink would have a zero wavelength, which would correspond to an infinite momentum and an infinite kinetic energy, and that would violate conservation of energy. But the kink in this example occurs at $r = 0$, which is right on top of the proton, where the electrical energy $-ke^2/r$ is infinite and *negative*. Since the electrical energy is negative and infinite, we're actually *required* to have an infinite positive kinetic energy in order to come up with a total that conserves energy.

Page 892: If you trace a circle going around the center, you run into a series of eight complete wavelengths. Its angular momentum is $8\hbar$.

Page 896: $n = 3$, $\ell = 0$, $\ell_z = 0$: one state; $n = 3$, $\ell = 1$, $\ell_z = -1$, 0, or 1: three states; $n = 3$, $\ell = 2$, $\ell_z = -2$, -1, 0, 1, or 2: five states

Answers

Answers for Chapter 2

Page 124, problem 37: $K = k_1k_2/(k_1 + k_2) = 1/(1/k_1 + 1/k_2)$

Answers for Chapter 3

Page 218, problem 5: After the collision it is moving at 1/3 of its initial speed, in the same direction it was initially going (it "follows through").

Page 225, problem 41: $Q = 1/\sqrt{2}$

Page 225, problem 43: (a) 7×10^{-8} radians, or about 4×10^{-6} degrees.

Page 227, problem 51: (a) $R = (2v^2/g)\sin\theta\cos\theta$ (c) $45\ ^\circ$

Page 227, problem 52: (a) The optimal angle is about 40°, and the resulting range is about 124 meters, which is about the length of a home run. (b) It goes about 9 meters farther. For comparison with reality, the stadium's web site claims a home run goes about 11 meters farther there than in a sea-level stadium.

Answers for Chapter 5

Page 339, problem 7: (c) $n \approx 16$

Page 339, problem 9: (a) $\sim 2-10\%$ (b) 5% (c) The high end for the body's actual efficiency is higher than the limit imposed by the laws of thermodynamics. However, the high end of the 1-5 watt range quoted in the problem probably includes large people who aren't just lying around. Still, it's impressive that the human body comes so close to the thermodynamic limit.

Page 340, problem 10: (a) Looking up the relevant density for air, and converting everything to mks, we get a frequency of 730 Hz. This is on the right order of magnitude, which is promising, considering the crudeness of the approximation. (b) This brings the result down to 400 Hz, which is amazingly close to the observed frequency of 300 Hz.

Answers for Chapter 6

Page 382, problem 9: (b) $g/2$

Page 383, problem 13: Check: The actual length of a flute is about 66 cm from the tip of the mouthpiece to the end of the bell.

Page 382, problem 8: (a) $T = \mu\omega^2 r^2$

Page 384, problem 17: (a) $f = 4\alpha/(1+\alpha)^2$ (b) $v_2 = \sqrt{v_1 v_3}$

Answers for Chapter 10

Page 642, problem 22: (a) $E = 2k\lambda/R$.

Answers for Chapter 11

Page 724, problem 5: (a) $I = \lambda v$.

Page 725, problem 6: (b) $2kI_1I_2L/c^2R$.

Answers for Chapter 12

Page 819, problem 61: f/ϵ

Answers for Chapter 13

Page 923, problem 40: about 10^{-34}

Solutions

Solutions for Chapter 0

Page 48, problem 6:

$$134\ \text{mg} \times \frac{10^{-3}\ \text{g}}{1\ \text{mg}} \times \frac{10^{-3}\ \text{kg}}{1\ \text{g}} = 1.34 \times 10^{-4}\ \text{kg}$$

Page 48, problem 8: (a) Let's do 10.0 g and 1000 g. The arithmetic mean is 505 grams. It comes out to be 0.505 kg, which is consistent. (b) The geometric mean comes out to be 100 g or 0.1 kg, which is consistent. (c) If we multiply meters by meters, we get square meters. Multiplying grams by grams should give square grams! This sounds strange, but it makes sense. Taking the square root of square grams (g^2) gives grams again. (d) No. The superduper mean of two quantities with units of grams wouldn't even be something with units of grams! Related to this shortcoming is the fact that the superduper mean would fail the kind of consistency test carried out in the first two parts of the problem.

Page 49, problem 12: (a) They're all defined in terms of the ratio of side of a triangle to another. For instance, the tangent is the length of the opposite side over the length of the adjacent side. Dividing meters by meters gives a unitless result, so the tangent, as well as the other trig functions, is unitless. (b) The tangent function gives a unitless result, so the units on the right-hand side had better cancel out. They do, because the top of the fraction has units of meters squared, and so does the bottom.

Page 50, problem 17: The problem requires us to relate a and t, for a fixed value of the distance Δx. To find a relationship among these three variables, we start with $\mathrm{d}^2 x/\mathrm{d}t^2 = a$, and integrate twice to find $\Delta x = \frac{1}{2}at^2$. This tells us that for a fixed value of Δx, we have $t \propto 1/\sqrt{a}$. Decreasing a by a factor of 3 means that t will increase by a factor of $\sqrt{3} = 1.7$. (The given piece of data, 3, only has one sig fig, but rounding the final result off to one sig fig, giving 2 rather then 1.7, would be a little too severe. As discussed in section 0.1.10, sig figs are only a rule of thumb, and when in doubt, you can change the input data to see how much the output would have changed. The ratio of the gravitational fields on Earth and Mars must be in the range from 2.5 to 3.5, since otherwise the given data would not have been rounded off to 3. Using this range of inputs, the possible range of values for the final result becomes 1.6 to 1.9. The final digit in the 1.7 is therefore a little uncertain, but it's not complete garbage. It carries useful information, and should not be thrown out.)

Page 51, problem 19: (a) Solving for $\Delta x = \frac{1}{2}at^2$ for a, we find $a = 2\Delta x/t^2 = 5.51\ \mathrm{m/s^2}$. (b) $v = \sqrt{2a\Delta x} = 66.6$ m/s. (c) The actual car's final velocity is less than that of the idealized constant-acceleration car. If the real car and the idealized car covered the quarter mile in the same time but the real car was moving more slowly at the end than the idealized one, the real car must have been going faster than the idealized car at the beginning of the race. The real car apparently has a greater acceleration at the beginning, and less acceleration at the end. This make sense, because every car has some maximum speed, which is the speed beyond which it cannot accelerate.

Page 52, problem 31:

$$1\ \mathrm{mm}^2 \times \left(\frac{1\ \mathrm{cm}}{10\ \mathrm{mm}}\right)^2 = 10^{-2}\ \mathrm{cm}^2$$

Page 52, problem 32: The bigger scope has a diameter that's ten times greater. Area scales as the square of the linear dimensions, so its light-gathering power is a hundred times greater (10×10).

Page 52, problem 33: Since they differ by two steps on the Richter scale, the energy of the bigger quake is 10000 times greater. The wave forms a hemisphere, and the surface area of the hemisphere over which the energy is spread is proportional to the square of its radius. If the amount of vibration was the same, then the surface areas much be in the ratio of 10000:1, which means that the ratio of the radii is 100:1.

Page 53, problem 38: The cone of mixed gin and vermouth is the same shape as the cone of vermouth, but its linear dimensions are doubled, so its volume is 8 times greater. The ratio of gin to vermouth is 7 to 1.

Page 53, problem 40: Scaling down the linear dimensions by a factor of 1/10 reduces the volume by a factor of $(1/10)^3 = 1/1000$, so if the whole cube is a liter, each small one is one milliliter.

Page 54, problem 41: Let's estimate the Great Wall's volume, and then figure out how many bricks that would represent. The wall is famous because it covers pretty much all of China's northern border, so let's say it's 1000 km long. From pictures, it looks like it's about 10 m high and 10 m wide, so the total volume would be $10^6\ \text{m} \times 10\ \text{m} \times 10\ \text{m} = 10^8\ \text{m}^3$. If a single brick has a volume of 1 liter, or $10^{-3}\ \text{m}^3$, then this represents about 10^{11} bricks. If one person can lay 10 bricks in an hour (taking into account all the preparation, etc.), then this would be 10^{10} man-hours.

Page 54, problem 44: Directly guessing the number of jelly beans would be like guessing volume directly. That would be a mistake. Instead, we start by estimating the linear dimensions, in units of beans. The contents of the jar look like they're about 10 beans deep. Although the jar is a cylinder, its exact geometrical shape doesn't really matter for the purposes of our order-of-magnitude estimate. Let's pretend it's a rectangular jar. The horizontal dimensions are also something like 10 beans, so it looks like the jar has about $10 \times 10 \times 10$ or $\sim 10^3$ beans inside.

Solutions for Chapter 1

Page 71, problem 12: To the person riding the moving bike, bug A is simply going in circles. The only difference between the motions of the two wheels is that one is traveling through space, but motion is relative, so this doesn't have any effect on the bugs. It's equally hard for each of them.

Solutions for Chapter 2

Page 118, problem 1: (a) The energy stored in the gasoline is being changed into heat via frictional heating, and also probably into sound and into energy of water waves. Note that the kinetic energy of the propeller and the boat are not changing, so they are not involved in the energy transformation. (b) The crusing speed would be greater by a factor of the cube root of 2, or about a 26% increase.

Page 118, problem 2: We don't have actual masses and velocities to plug in to the equation, but that's OK. We just have to reason in terms of ratios and proportionalities. Kinetic energy is proportional to mass and to the square of velocity, so B's kinetic energy equals $(13.4\ \text{J})(3.77)/(2.34)^2 = 9.23\ \text{J}$.

Page 118, problem 3: Room temperature is about 20°C. The fraction of the power that actually goes into heating the water is

$$\frac{(250\ \text{g})/(0.24\ \text{J/g}^\circ\text{C}) \times (100^\circ\text{C-}20^\circ\text{C})/126\ \text{s}}{1.25 \times 10^3\ \text{J/s}} = 0.53$$

So roughly half of the energy is wasted. The wasted energy might be in several forms: heating of the cup, heating of the oven itself, or leakage of microwaves from the oven.

Page 118, problem 5:

$$\begin{aligned}
E_{total,i} &= E_{total,f} \\
U_i + heat_i &= U_f + heat_f + K_f \\
\frac{1}{2}mv^2 &= U_i - U_f + heat_i - heat_f \\
&= -\Delta U - \Delta heat \\
v &= \sqrt{2\left(\frac{-\Delta U - \Delta heat}{m}\right)} \\
&= 6.4 \text{ m/s}
\end{aligned}$$

Solutions for Chapter 3

Page 218, problem 4: A conservation law is about addition: it says that when you add up a certain thing, the total always stays the same. Funkosity would violate the additive nature of conservation laws, because a two-kilogram mass would have twice as much funkosity as a pair of one-kilogram masses moving at the same speed.

Page 219, problem 12: Momentum is a vector. The total momentum of the molecules is always zero, since the momenta in different directions cancal out on the average. Cooling changes individual molecular momenta, but not the total.

Page 220, problem 15: $a = \Delta v/\Delta t$, and also $a = F/m$, so

$$\begin{aligned}
\Delta t &= \frac{\Delta v}{a} \\
&= \frac{m\Delta v}{F} \\
&= \frac{(1000 \text{ kg})(50 \text{ m/s} - 20 \text{ m/s})}{3000 \text{ N}} \\
&= 10 \text{ s}
\end{aligned}$$

Page 221, problem 23: (a) This is a measure of the box's resistance to a change in its state of motion, so it measures the box's mass. The experiment would come out the same in lunar gravity.
(b) This is a measure of how much gravitational force it feels, so it's a measure of weight. In lunar gravity, the box would make a softer sound when it hit.
(c) As in part a, this is a measure of its resistance to a change in its state of motion: its mass. Gravity isn't involved at all.

Page 223, problem 34: (a) The swimmer's acceleration is caused by the water's force on the swimmer, and the swimmer makes a backward force on the water, which accelerates the water backward. (b) The club's normal force on the ball accelerates the ball, and the ball makes a backward normal force on the club, which decelerates the club. (c) The bowstring's normal force accelerates the arrow, and the arrow also makes a backward normal force on the string. This force on the string causes the string to accelerate less rapidly than it would if the bow's force was the only one acting on it. (d) The tracks' backward frictional force slows the locomotive down. The locomotive's forward frictional force causes the whole planet earth to accelerate by a tiny amount, which is too small to measure because the earth's mass is so great.

Page 224, problem 37: (a) Spring constants in parallel add, so the spring constant has to be proportional to the cross-sectional area. Two springs in series give half the spring constant, three springs in series give 1/3, and so on, so the spring constant has to be inversely proportional to the length. Summarizing, we have $k \propto A/L$.
(b) With the Young's modulus, we have $k = (A/L)E$.The spring constant has units of N/m, so the units of E would have to be N/m^2.

Page 226, problem 44: By conservation of momentum, the total momenta of the pieces after the explosion is the same as the momentum of the firework before the explosion. However, there is no law of conservation of kinetic energy, only a law of conservation of energy. The chemical energy in the gunpowder is converted into heat and kinetic energy when it explodes. All we can say about the kinetic energy of the pieces is that their total is greater than the kinetic energy before the explosion.

Page 226, problem 45: Let m be the mass of the little puck and $M = 2.3m$ be the mass of the big one. All we need to do is find the direction of the total momentum vector before the collision, because the total momentum vector is the same after the collision. Given the two components of the momentum vector $p_x = mv$ and $p_y = Mv$, the direction of the vector is $\tan^{-1}(p_y/p_x) = 23°$ counterclockwise from the big puck's original direction of motion.

Page 229, problem 62: We want to find out about the velocity vector $\mathbf{v}_{BG}$ of the bullet relative to the ground, so we need to add Annie's velocity relative to the ground $\mathbf{v}_{AG}$ to the bullet's velocity vector $\mathbf{v}_{BA}$ relative to her. Letting the positive x axis be east and y north, we have

$$\begin{aligned} v_{BA,x} &= (140 \text{ mi/hr}) \cos 45° \\ &= 100 \text{ mi/hr} \\ v_{BA,y} &= (140 \text{ mi/hr}) \sin 45° \\ &= 100 \text{ mi/hr} \end{aligned}$$

and

$$\begin{aligned} v_{AG,x} &= 0 \\ v_{AG,y} &= 30 \text{ mi/hr}. \end{aligned}$$

The bullet's velocity relative to the ground therefore has components

$$v_{BG,x} = 100 \text{ mi/hr}$$

and

$$v_{BG,y} = 130 \text{ mi/hr}.$$

Its speed on impact with the animal is the magnitude of this vector

$$\begin{aligned} |\mathbf{v}_{BG}| &= \sqrt{(100 \text{ mi/hr})^2 + (130 \text{ mi/hr})^2} \\ &= 160 \text{ mi/hr} \end{aligned}$$

(rounded off to two significant figures).

Page 229, problem 63: Since its velocity vector is constant, it has zero acceleration, and the sum of the force vectors acting on it must be zero. There are three forces acting on the plane: thrust, lift, and gravity. We are given the first two, and if we can find the third we can infer the plane's mass. The sum of the y components of the forces is zero, so

$$\begin{aligned} 0 &= F_{thrust,y} + F_{lift,y} + F_{g,y} \\ &= |\mathbf{F}_{thrust}| \sin\theta + |\mathbf{F}_{lift}| \cos\theta - mg. \end{aligned}$$

The mass is

$$\begin{aligned} m &= (|\mathbf{F}_{thrust}| \sin\theta + |\mathbf{F}_{lift}| \cos\theta)/g \\ &= 7.0 \times 10^4 \text{ kg}. \end{aligned}$$

Page 229, problem 64: (a) Since the wagon has no acceleration, the total forces in both the x and y directions must be zero. There are three forces acting on the wagon: T, $\mathbf{F}_g$, and the normal force from the ground, $\mathbf{F}_n$. If we pick a coordinate system with x being horizontal and y vertical, then the angles of these forces measured counterclockwise from the x axis are $90° - \phi$, $270°$, and $90° + \theta$, respectively. We have

$$\begin{aligned} F_{x,total} &= T\cos(90° - \phi) + F_g \cos(270°) + F_n cos(90° + \theta) \\ F_{y,total} &= T\sin(90° - \phi) + F_g \sin(270°) + F_n sin(90° + \theta), \end{aligned}$$

which simplifies to

$$\begin{aligned} 0 &= T\sin\phi - F_n \sin\theta \\ 0 &= T\cos\phi - F_g + F_n \cos\theta. \end{aligned}$$

The normal force is a quantity that we are not given and do not wish to find, so we should choose it to eliminate. Solving the first equation for $F_n = (\sin\phi/\sin\theta)T$, we eliminate F_n from the second equation,

$$0 = T\cos\phi - F_g + T\sin\phi\cos\theta/\sin\theta$$

and solve for T, finding

$$T = \frac{F_g}{\cos\phi + \sin\phi\cos\theta/\sin\theta}$$

Multiplying both the top and the bottom of the fraction by $\sin\theta$, and using the trig identity for $\sin(\theta + \phi)$ gives the desired result,

$$T = \frac{\sin\theta}{\sin(\theta + \phi)} F_g s$$

(b) The case of $\phi = 0$, i.e. pulling straight up on the wagon, results in $T = F_g$: we simply support the wagon and it glides up the slope like a chair-lift on a ski slope. In the case of $\phi = 180° - \theta$, T becomes infinite. Physically this is because we are pulling directly into the ground, so no amount of force will suffice.

Page 230, problem 65: (a) If there was no friction, the angle of repose would be zero, so the coefficient of static friction, μ_s, will definitely matter. We also make up symbols θ, m and g for the angle of the slope, the mass of the object, and the acceleration of gravity. The forces form a triangle just like the one in example 68 on page 203, but instead of a force applied by an external object, we have static friction, which is less than $\mu_s F_n$. As in that example, $F_s = mg\sin\theta$, and $F_s < \mu_s F_n$, so

$$mg\sin\theta < \mu_s F_n.$$

From the same triangle, we have $F_n = mgcos\theta$, so

$$mg\sin\theta < \mu_s mg\cos\theta.$$

Rearranging,

$$\theta < \tan^{-1}\mu_s.$$

(b) Both m and g canceled out, so the angle of repose would be the same on an asteroid.

Solutions for Chapter 4

Page 288, problem 1: The pliers are not moving, so their angular momentum remains constant at zero, and the total torque on them must be zero. Not only that, but each half of the pliers must have zero total torque on it. This tells us that the magnitude of the torque at one end must be the same as that at the other end. The distance from the axis to the nut is about 2.5 cm, and the distance from the axis to the centers of the palm and fingers are about 8 cm. The angles are close enough to 90°that we can pretend they're 90 degrees, considering the rough nature of the other assumptions and measurements. The result is (300 N)(2.5 cm)=(F)(8 cm), or F=90 N.

Page 294, problem 37: The foot of the rod is moving in a circle relative to the center of the rod, with speed $v = \pi b/T$, and acceleration $v^2/(b/2) = (\pi^2/8)g$. This acceleration is initially upward, and is greater in magnitude than g, so the foot of the rod will lift off without dragging. We could also worry about whether the foot of the rod would make contact with the floor again before the rod finishes up flat on its back. This is a question that can be settled by graphing, or simply by inspection of figure i on page 276. The key here is that the two parts of the acceleration are both independent of m and b, so the result is univeral, and it does suffice to check a graph in a single example. In practical terms, this tells us something about how difficult the trick is to do. Because $\pi^2/8 = 1.23$ isn't much greater than unity, a hit that is just a little too weak (by a factor of $1.23^{1/2} = 1.11$) will cause a fairly obvious qualitative change in the results. This is easily observed if you try it a few times with a pencil.

Solutions for Chapter 5

Page 340, problem 11: (a) We have

$$\begin{aligned}\mathrm{d}P &= \rho g\,\mathrm{d}y\\ \Delta P &= \int \rho g\,\mathrm{d}y,\end{aligned}$$

and since we're taking water to be incompressible, and g doesn't change very much over 11 km of height, we can treat ρ and g as constants and take them outside the integral.

$$\begin{aligned}\Delta P &= \rho g \Delta y\\ &= (1.0\ \mathrm{g/cm^3})(9.8\ \mathrm{m/s^2})(11.0\ \mathrm{km})\\ &= (1.0\times 10^3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(1.10\times 10^4\ \mathrm{m})\\ &= 1.0\times 10^8\ \mathrm{Pa}\\ &= 1.0\times 10^3\ \mathrm{atm}.\end{aligned}$$

The precision of the result is limited to a few percent, due to the compressibility of the water, so we have at most two significant figures. If the change in pressure were exactly a thousand atmospheres, then the pressure at the bottom would be 1001 atmospheres; however, this distinction is not relevant at the level of approximation we're attempting here.
(b) Since the air in the bubble is in thermal contact with the water, it's reasonable to assume that it keeps the same temperature the whole time. The ideal gas law is $PV = nkT$, and rewriting this as a proportionality gives

$$V \propto P^{-1},$$

or

$$\frac{V_f}{V_i} = \left(\frac{P_f}{P_i}\right)^{-1} \approx 10^3.$$

Since the volume is proportional to the cube of the linear dimensions, the growth in radius is about a factor of 10.

Page 340, problem 12: (a) Roughly speaking, the thermal energy is $\sim k_B T$ (where k_B is the Boltzmann constant), and we need this to be on the same order of magnitude as ke^2/r (where k is the Coulomb constant). For this type of rough estimate it's not especially crucial to get all the factors of two right, but let's do so anyway. Each proton's average kinetic energy due to motion along a particular axis is $(1/2)k_B T$. If two protons are colliding along a certain line in the center-of-mass frame, then their average combined kinetic energy due to motion along that axis is $2(1/2)k_B T = k_B T$. So in fact the factors of 2 cancel. We have $T = ke^2/k_B r$.
(b) The units are $\mathrm{K} = (\mathrm{J{\cdot}m/C^2})(\mathrm{C^2})/((\mathrm{J/K}){\cdot}\mathrm{m})$, which does work out.
(c) The numerical result is $\sim 10^{10}$ K, which as suggested is much higher than the temperature at the core of the sun.

Page 341, problem 13: If the full-sized brick A undergoes some process, such as heating it with a blowtorch, then we want to be able to apply the equation $\Delta S = Q/T$ to either the whole brick or half of it, which would be identical to B. When we redefine the boundary of the system to contain only half of the brick, the quantities ΔS and Q are each half as big, because entropy and energy are additive quantities. T, meanwhile, stays the same, because temperature isn't additive — two cups of coffee aren't twice as hot as one. These changes to the variables leave the equation consistent, since each side has been divided by 2.

Page 341, problem 14: (a) If the expression $1 + by$ is to make sense, then by has to be unitless, so b has units of m^{-1}. The input to the exponential function also has to be unitless, so k also has of m^{-1}. The only factor with units on the right-hand side is P_o, so P_o must have units of pressure, or Pa.
(b)

$$\begin{aligned} \mathrm{d}P &= \rho g \, \mathrm{d}y \\ \rho &= \frac{1}{g}\frac{\mathrm{d}P}{\mathrm{d}y} \\ &= \frac{P_o}{g} e^{-ky}(-k - kby + b) \end{aligned}$$

(c) The three terms inside the parentheses on the right all have units of m^{-1}, so it makes sense to add them, and the factor in parentheses has those units. The units of the result from b then look like

$$\begin{aligned}\frac{\mathrm{kg}}{\mathrm{m}^3} &= \frac{\mathrm{Pa}}{\mathrm{m/s}^2}\mathrm{m}^{-1}\\ &= \frac{\mathrm{N/m}^2}{\mathrm{m}^2/\mathrm{s}^2}\\ &= \frac{\mathrm{kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{s}^{-2}}{\mathrm{m}^2/\mathrm{s}^2},\end{aligned}$$

which checks out.

Solutions for Chapter 7

Page 447, problem 17: (a) Plugging in, we find

$$\sqrt{\frac{1-w}{1+w}} = \sqrt{\frac{1-u}{1+u}}\sqrt{\frac{1-v}{1+v}}.$$

(b) First let's simplify by squaring both sides.

$$\frac{1-w}{1+w} = \frac{1-u}{1+u}\cdot\frac{1-v}{1+v}.$$

For convenience, let's write A for the right-hand side of this equation. We then have

$$\begin{gathered}\frac{1-w}{1+w} = A\\ 1-w = A+Aw.\end{gathered}$$

Solving for w,

$$\begin{aligned}w &= \frac{1-A}{1+A}\\ &= \frac{(1+u)(1+v)-(1-u)(1-v)}{(1+u)(1+v)+(1-u)(1-v)}\\ &= \frac{2(u+v)}{2(1+uv)}\\ &= \frac{u+v}{1+uv}\end{aligned}$$

(c) This is all in units where $c=1$. The correspondence principle says that we should get $w \approx u+v$ when both u and v are small compared to 1. Under those circumstances, uv is the product of two very small numbers, which makes it very, very small. Neglecting this term in the denominator, we recover the nonrelativistic result.

Page 447, problem 18: Among the spacelike vectors, **a** and **e** are clearly congruent, because they're the same except for a rotation in space; this is the same as the definition of congruence in ordinary Euclidean geometry, where rotation doesn't matter. Vector **b** is also congruent to these, since it represents an interval $3^2-5^2=-4^2$, just like the other two.

The lightlike vectors **c** and **d** both represent intervals of zero, so they're congruent, even though **c** is a double-scale version of **d**.

The timelike vectors **f** and **g** are not congruent to each other or to any of the others; **f** represents an interval of 2^2, while **g**'s interval is 4^2.

Page 448, problem 22: At the center of each of the large triangle's sides, the angles add up to 180° because they form a straight line. Therefore $4s = S+3\times 180°$, so $S-180° = 4(s-180°)$.

Page 449, problem 28: By the equivalence principle, we can adopt a frame tied to the tossed clock, B, and in this frame there is no gravitational field. We see a desk and clock A go by. The desk applies a force to clock A, decelerating it and then reaccelerating it so that it comes back. We've already established that the effect of motion is to slow down time, so clock A reads a smaller time interval.

Hints for Chapter 8

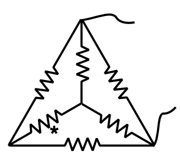
Page 512, problem 15: The force on the lithium ion is the vector sum of all the forces of all the quadrillions of sodium and chlorine atoms, which would obviously be too laborious to calculate. Nearly all of these forces, however, are canceled by a force from an ion on the opposite side of the lithium.

Solutions for Chapter 9

Page 550, problem 1: $\Delta t = \Delta q/I = e/I = 0.16\ \mu\text{s}$

Page 551, problem 12: In series, they give 11 kΩ. In parallel, they give $(1/1\ \text{k}\Omega + 1/10\ \text{k}\Omega)^{-1} = 0.9\ \text{k}\Omega$.

Page 554, problem 25: The actual shape is irrelevant; all we care about is what's connected to what. Therefore, we can draw the circuit flattened into a plane. Every vertex of the tetrahedron is adjacent to every other vertex, so any two vertices to which we connect will give the same resistance. Picking two arbitrarily, we have this:



This is unfortunately a circuit that cannot be converted into parallel and series parts, and that's what makes this a hard problem! However, we can recognize that by symmetry, there is zero current in the resistor marked with an asterisk. Eliminating this one, we recognize the whole arrangement as a triple parallel circuit consisting of resistances R, $2R$, and $2R$. The resulting resistance is $R/2$.

Page 555, problem 29: (a) Conservation of energy gives

$$\begin{aligned}
U_A &= U_B + K_B \\
K_B &= U_A - U_B \\
\frac{1}{2}mv^2 &= e\Delta V \\
v &= \sqrt{\frac{2e\Delta V}{m}}
\end{aligned}$$

(b) Plugging in numbers, we get 5.9×10^7 m/s. This is about 20% of the speed of light, so the nonrelativistic assumption was good to at least a rough approximation.

Page 556, problem 32: It's much more practical to measure voltage differences. To measure a current, you have to break the circuit somewhere and insert the meter there, but it's not possible to disconnect the circuits sealed inside the board.

Solutions for Chapter 10

Page 640, problem 16: By symmetry, the field is always directly toward or away from the center. We can therefore calculate it along the x axis, where $r = x$, and the result will be valid for any location at that distance from the center.

$$\begin{aligned} E &= -\frac{\mathrm{d}V}{\mathrm{d}x} \\ &= -\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{-1}e^{-x}\right) \\ &= x^{-2}e^{-x} + x^{-1}e^{-x} \end{aligned}$$

At small x, near the proton, the first term dominates, and the exponential is essentially 1, so we have $E \propto x^{-2}$, as we expect from the Coulomb force law. At large x, the second term dominates, and the field approaches zero faster than an exponential.

Page 648, problem 56:

$$\begin{aligned} \sin(a+b) &= \left(e^{i(a+b)} - e^{-i(a+b)}\right)/2i \\ &= \left(e^{ia}e^{ib} - e^{-ia}e^{-ib}\right)/2i \\ &= \left[(\cos a + i\sin a)(\cos b + i\sin b) - (\cos a - i\sin a)(\cos b - i\sin b)\right]/2i \\ &= \cos a\sin b + \sin a\cos b \end{aligned}$$

By a similar computation, we find $\cos(a+b) = \cos a\cos b - \sin a\sin b$.

Page 648, problem 57: If $z^3 = 1$, then we know that $|z| = 1$, since cubing z cubes its magnitude. Cubing z triples its argument, so the argument of z must be a number that, when tripled, is equivalent to an angle of zero. There are three possibilities: $0\times 3 = 0$, $(2\pi/3)\times 3 = 2\pi$, and $(4\pi/3)\times 3 = 4\pi$. (Other possibilities, such as $(32\pi/3)$, are equivalent to one of these.) The solutions are:

$$z = 1,\ e^{2\pi i/3},\ e^{4\pi i/3}$$

Page 648, problem 58: We can think of this as a polynomial in x or a polynomial in y — their roles are symmetric. Let's call x the variable. By the fundamental theorem of algebra, it must be possible to factor it into a product of three linear factors, if the coefficients are allowed to be complex. Each of these factors causes the product to be zero for a certain value of x. But the condition for the expression to be zero is $x^3 = y^3$, which basically means that the ratio of x to y must be a third root of 1. The problem, then, boils down to finding the three third roots of 1, as in problem 57. Using the result of that problem, we find that there are zeroes when x/y equals 1, $e^{2\pi i/3}$, and $e^{4\pi i/3}$. This tells us that the factorization is $(x-y)(x-e^{2\pi i/3}y)(x-e^{4\pi i/3}y)$.

The second part of the problem asks us to factorize as much as possible using real coefficients. Our only hope of doing this is to multiply out the two factors that involve complex coefficients, and see if they produce something real. In fact, we can anticipate that it will work, because the coefficients are complex conjugates of one another, and when a quadratic has two complex roots, they are conjugates. The result is $(x-y)(x^2+xy+y^2)$.

Solutions for Chapter 11

Page 735, problem 51: (a) For a material object, $\mathbf{p} = m\mathbf{v}$. The velocity vector reverses itself, but mass is still positive, so the momentum vector is reversed.
(b) In the forward-time universe, conservation of momentum is $\mathbf{p}_{1,i} + \mathbf{p}_{2,i} = \mathbf{p}_{1,f} + \mathbf{p}_{2,f}$. In the

backward-time universe, all the momenta are reversed, but that just negates both sides of the equation, so momentum is still conserved.

Page 737, problem 54: Note that in the Biot-Savart law, the variable $\mathbf{r}$ is defined as a vector that points from the current to the point at which the field is being calculated, whereas in the polar coordinates used to express the equation of the spiral, the vector more naturally points the opposite way. This requires some fiddling with signs, which I'll suppress, and simply identify $\mathrm{d}\boldsymbol{\ell}$ with $\mathrm{d}\mathbf{r}$.

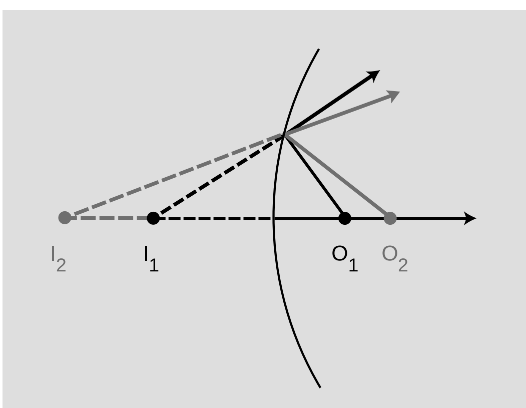
$$\mathbf{B} = \frac{kI}{c^2}\int \frac{\mathrm{d}\boldsymbol{\ell}\times\mathbf{r}}{r^3}$$

The vector $\mathrm{d}\mathbf{r}$ has components $\mathrm{d}x = w(\cos\theta - \theta\sin\theta)$ and $\mathrm{d}y = w(\sin\theta + \theta\cos\theta)$. Evaluating the vector cross product, and substituting θ/w for r, we find

$$\begin{aligned}\mathbf{B} &= \frac{kI}{c^2 w}\int \frac{\theta(\cos\theta\sin\theta - \theta\sin^2\theta - \cos\theta\sin\theta - \theta\cos^2\theta)\,\mathrm{d}\theta}{\theta^3}\\ &= \frac{kI}{c^2 w}\int\frac{\mathrm{d}\theta}{\theta}\\ &= \frac{kI}{c^2 w}\ln\frac{\theta_2}{\theta_1}\\ &= \frac{kI}{c^2 w}\ln\frac{b}{a}\end{aligned}$$

Solutions for Chapter 12

Page 807, problem 16: See the ray diagram below. Decreasing θ_o decreases θ_i, so the equation $\theta_f = \pm\theta_i + \pm\theta_o$ must have opposite signs on the right. Since θ_o is bigger than θ_i, the only way to get a positive θ_f is if the signs are $\theta_f = -\theta_i + \theta_o$. This gives $1/f = -1/d_i + 1/d_o$.



Page 807, problem 19: (a) The object distance is less than the focal length, so the image is virtual: because the object is so close, the cone of rays is diverging too strongly for the mirror to bring it back to a focus. (b) At an object distance of 30 cm, it's clearly going to be real. With the object distance of 20 cm, we're right at the crossing-point between real and virtual. For this object position, the reflected rays will be parallel. We could consider this to be an image at infinity. (c),(d) A diverging mirror can only make virtual images.

Page 811, problem 39: Since d_o is much greater than d_i, the lens-film distance d_i is essentially the same as f. (a) Splitting the triangle inside the camera into two right triangles, straightforward trigonometry gives

$$\theta = 2\tan^{-1}\frac{w}{2f}$$

for the field of view. This comes out to be 39° and 64° for the two lenses. (b) For small angles, the tangent is approximately the same as the angle itself, provided we measure everything in radians. The equation above then simplifies to

$$\theta = \frac{w}{f}$$

The results for the two lenses are .70 rad = 40°, and 1.25 rad = 72°. This is a decent approximation.

(c) With the 28-mm lens, which is closer to the film, the entire field of view we had with the 50-mm lens is now confined to a small part of the film. Using our small-angle approximation $\theta = w/f$, the amount of light contained within the same angular width θ is now striking a piece of the film whose linear dimensions are smaller by the ratio 28/50. Area depends on the square of the linear dimensions, so all other things being equal, the film would now be overexposed by a factor of $(50/28)^2 = 3.2$. To compensate, we need to shorten the exposure by a factor of 3.2.

Page 819, problem 59: One surface is curved outward and one inward. Therefore the minus sign applies in the lensmaker's equation. Since the radii of curvature are equal, the quantity $1/r_1 - 1/r_2$ equals zero, and the resulting focal length is infinite. A big focal length indicates a weak lens. An infinite focal length tells us that the lens is infinitely weak — it doesn't focus or defocus rays at all.

Solutions for Chapter 13

Page 925, problem 48: The expressions $|\Psi|^2$ and $|\Psi^2|$ are identical, because the magnitude of a product is the product of the magnitudes. These expressions give positive real numbers as their results, which makes sense for a probability density. The expression Ψ^2 need not be real, and if it is real, it may be negative. It cannot be interpreted as a probability density. As a concrete example, suppose that $\Psi = bi$, where b is a real number with units. Then $|\Psi|^2 = |\Psi^2| = b^2$, which is real and positive, but $\Psi^2 = -b^2$, which clearly can't be interpreted probabilistically, because it's negative.

Page 925, problem 49: (a) The quantity $x - y$ vanishes along the line $y = x$ lying in the first quadrant at a 45-degree angle between the axes. Squaring produces a trough parallel to this line, with a parabolic cross-section. Geometrically, the Laplacian can be interpreted as a measure of how much the value of f at a point differs from its average value on a small circle centered on that point. The trough is concave up, so we can predict that the Laplacian will be positive everywhere.
(b) The zero result is clearly wrong because it disagrees with our conclusion from part a that the Laplacian is positive. A correct calculation gives $\partial^2(x-y)^2/\partial x^2 + \partial^2(x-y)^2/\partial y^2 = 4$.
(c) If we rotate our coordinate axes counterclockwise by 45 degrees, then we have a parabolic trough oriented along the x axis. In terms of these new coordinates, $\partial f/\partial x = 0$, while $\partial f/\partial y$ is nonzero almost everywhere.
Remark: The mistake described in the question is a common one, and is apparently based on the idea that the notation ∇^2 must mean applying an operator ∇ twice. For those with some exposure to vector calculus, it may be of interest to note that the Laplacian *is* equivalent to the divergence of the gradient, which can be notated either div(grad f) or $\nabla \cdot (\nabla f)$. The important thing to recognize is that the gradient, notated grad f or ∇f, outputs a *vector*, not a scalar like the quantity Q defined in this problem.

Appendix 5: Useful Data

.0.3 Notation and terminology, compared with other books

Almost all the notation and terminology in *Simple Nature* is standard, but there are some cases where there is no universal standard, and a very few cases where I've intentionally deviated from a universal standard. The notation used by physicists is also different from that used by electrical and mechanical engineers; I use physics terminology and notation (notably $\sqrt{-1} = i$, not j, and "torque" rather than "moment"), but employ the SI system of units used in engineering, rather than the cgs units favored by some physicists.

Nonstandard terminology:

Potential energy is referred to in this book as *interaction energy*, or according to its type: *gravitational energy*, *electrical energy*, etc.

The potential, in an electrical context, is referred to as *voltage*, e.g. I say that $V = kq/r$ is the voltage surrounding a point charge.

Heat and thermal energy are both referred to as *heat.* This is in keeping with casual usage among scientists, but formal written usage dictates the use of "thermal energy" to mean the kinetic energy an object has because of its molecules' random motion, while "heat" is the transfer of thermal energy.

Notation for which there is no universal standard:

Kinetic energy is written K. Standard notation is K, T, or KE.

Interaction energy is written U. Standard notation is U, V, or PE.

The unit vectors are $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$. Standard notation is either $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ or $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$.

Distance from an axis in cylindrical coordinates is R. A more common notation in math books is ρ, but this would conflict with the standard physics notation for the charge density.

Vibrations do not have very well standardized terminology or notation. I use "frequency" to refer to both f and ω, depending on the context to make it clear which is meant. The frequency of free, damped oscillations is ω_f, which is only approximately the same as $\omega_o = \sqrt{k/m}$. The full width at half-maximum of the resonance peak (on a plot of energy versus frequency) is $\Delta\omega$.

The coupling constants for electricity and magnetism are written as k and k/c^2. This is standard notation, but it would be more common in SI calculations to see everything expressed in terms of $\epsilon_o = 1/4\pi k$ and $\mu_o = 4\pi k/c^2$. Numerically, we have $k = 8.99 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$ and $k/c^2 = 10^{-7}\ \mathrm{N/A^2}$, the latter being an exact relation.

.0.4 Notation and units

quantity	unit	symbol
distance	meter, m	$x, \Delta x$
time	second, s	$t, \Delta t$
mass	kilogram, kg	m
density	kg/m^3	ρ
force	newton, $1\ \text{N}=1\ \text{kg}\cdot\text{m/s}^2$	F
velocity	m/s	v
acceleration	m/s^2	a
gravitational field	$\text{J/kg}\cdot\text{m}$ or m/s^2	g
energy	joule, J	E (also electric field)
momentum	$\text{kg}\cdot\text{m/s}$	p
angular momentum	$\text{kg}\cdot\text{m}^2\text{/s}$ or $\text{J}\cdot\text{s}$	L (also inductance)
power	watt, 1 W = 1 J/s	P (also pressure)
pressure	$1\ \text{Pa}=1\ \text{N/m}^2$	P (also power)
temperature	K	T (also period)
period	s	T (also temperature)
wavelength	m	λ
frequency	s^{-1} or Hz	f
charge	coulomb, C	q
voltage	volt, 1 V = 1 J/C	V
current	ampere, 1 A = 1 C/s	I
resistance	ohm, $1\ \Omega = 1$ V/A	R
capacitance	farad, 1 F = 1 C/V	C
inductance	henry, $1\ \text{H} = 1\ \text{V}\cdot\text{s/A}$	L (also angular momentum)
electric field	V/m or N/C	E (also energy)
magnetic field	tesla, $1\ \text{T} = 1\ \text{N}\cdot\text{s/C}\cdot\text{m}$	B
focal length	m	f
magnification	unitless	M
index of refraction	unitless	n
electron wavefunction	$\text{m}^{-3/2}$	Ψ

.0.5 Fundamental constants

gravitational constant	$G = 6.67 \times 10^{-11}\ \text{J}\cdot\text{m/kg}^2$
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K
Coulomb constant	$k = 8.99 \times 10^{9}\ \text{J}\cdot\text{m/C}^2$ or $\text{N}\cdot\text{m}^2/\text{C}^2$
quantum of charge	$e = 1.60 \times 10^{-19}$ C
speed of light	$c = 3.00 \times 10^{8}$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$

Note the use of the same notation, k, for both the Boltzmann constant and the Coulomb constant.

.0.6 Metric prefixes

M-	mega-	10^6
k-	kilo-	10^3
m-	milli-	10^{-3}
μ- (Greek mu)	micro-	10^{-6}
n-	nano-	10^{-9}
p-	pico-	10^{-12}
f-	femto-	10^{-15}

Note that the exponents go in steps of three. The exception is centi-, 10^{-2}, which is used only in the centimeter, and this doesn't require memorization, because a cent is 10^{-2} dollars.

.0.7 Nonmetric units

Nonmetric units in terms of metric ones:

1 inch	= 25.4 mm (by definition)
1 pound (lb)	= 4.5 newtons of force
1 scientific calorie	= 4.18 J
1 nutritional calorie	$= 4.18 \times 10^3$ J
1 gallon	$= 3.78 \times 10^3$ cm^3
1 horsepower	= 746 W

The pound is a unit of force, so it converts to newtons, not kilograms. A one-kilogram mass at the earth's surface experiences a gravitational force of $(1\ \text{kg})(9.8\ \text{m/s}^2) = 9.8\ \text{N} = 2.2\ \text{lb}$. The nutritional information on food packaging typically gives energies in units of calories, but those so-called calories are really kilocalories.

Relationships among U.S. units:

1 foot (ft)	= 12 inches
1 yard (yd)	= 3 feet
1 mile (mi)	= 5280 feet
1 ounce (oz)	= 1/16 pound

.0.8 The Greek alphabet

α	A	alpha	ι	I	iota	ρ	P	rho
β	B	beta	κ	K	kappa	σ	Σ	sigma
γ	Γ	gamma	λ	Λ	lambda	τ	T	tau
δ	Δ	delta	μ	M	mu	υ	Y	upsilon
ϵ	E	epsilon	ν	N	nu	ϕ	Φ	phi
ζ	Z	zeta	ξ	Ξ	xi	χ	X	chi
η	H	eta	o	O	omicron	ψ	Ψ	psi
θ	Θ	theta	π	Π	pi	ω	Ω	omega

.0.9 Subatomic particles

particle	mass (kg)	charge	radius (fm)
electron	9.109×10^{-31}	$-e$	$\lesssim 0.01$
proton	1.673×10^{-27}	$+e$	~ 1.1
neutron	1.675×10^{-27}	0	~ 1.1
neutrino	$\sim 10^{-39}$ kg ?	0	?

The radii of protons and neutrons can only be given approximately, since they have fuzzy surfaces. For comparison, a typical atom is about a million fm in radius.

.0.10 Earth, moon, and sun

body	mass (kg)	radius (km)	radius of orbit (km)
earth	5.97×10^{24}	6.4×10^{3}	1.49×10^{8}
moon	7.35×10^{22}	1.7×10^{3}	3.84×10^{5}
sun	1.99×10^{30}	7.0×10^{5}	—

.0.11 The periodic table

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La *	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac **	104 Rf	105 Ha	106	107	108	109	110	111	112	113	114	115	116	117	118

*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

.0.12 Atomic masses

These atomic masses are given in atomic mass units (u), where by definition the mass of an atom of the isotope carbon-12 equals 12 u. One atomic mass unit is the same as about 1.66×10^{-27} kg. Data are only given for naturally occurring elements.

Ag	107.9	Eu	152.0	Mo	95.9	Sc	45.0
Al	27.0	F	19.0	N	14.0	Se	79.0
Ar	39.9	Fe	55.8	Na	23.0	Si	28.1
As	74.9	Ga	69.7	Nb	92.9	Sn	118.7
Au	197.0	Gd	157.2	Nd	144.2	Sr	87.6
B	10.8	Ge	72.6	Ne	20.2	Ta	180.9
Ba	137.3	H	1.0	Ni	58.7	Tb	158.9
Be	9.0	He	4.0	O	16.0	Te	127.6
Bi	209.0	Hf	178.5	Os	190.2	Ti	47.9
Br	79.9	Hg	200.6	P	31.0	Tl	204.4
C	12.0	Ho	164.9	Pb	207.2	Tm	168.9
Ca	40.1	In	114.8	Pd	106.4	U	238
Ce	140.1	Ir	192.2	Pt	195.1	V	50.9
Cl	35.5	K	39.1	Pr	140.9	W	183.8
Co	58.9	Kr	83.8	Rb	85.5	Xe	131.3
Cr	52.0	La	138.9	Re	186.2	Y	88.9
Cs	132.9	Li	6.9	Rh	102.9	Yb	173.0
Cu	63.5	Lu	175.0	Ru	101.1	Zn	65.4
Dy	162.5	Mg	24.3	S	32.1	Zr	91.2
Er	167.3	Mn	54.9	Sb	121.8		

Appendix 6: Summary

Notation and units are summarized on page 966.

Chapter 0, Introduction and Review, page 13

Physics is the use of the scientific method to study the behavior of light and matter. The scientific method requires a cycle of theory and experiment, theories with both predictive and explanatory value, and reproducible experiments.

The metric system is a simple, consistent framework for measurement built out of the meter, the kilogram, and the second plus a set of prefixes denoting powers of ten. The most systematic method for doing conversions is shown in the following example:

$$370\ \text{ms} \times \frac{10^{-3}\ \text{s}}{1\ \text{ms}} = 0.37\ \text{s}$$

Mass is a measure of the amount of a substance. Mass can be defined gravitationally, by comparing an object to a standard mass on a double-pan balance, or in terms of inertia, by comparing the effect of a force on an object to the effect of the same force on a standard mass. The two definitions are found experimentally to be proportional to each other to a high degree of precision, so we usually refer simply to "mass," without bothering to specify which type.

A force is that which can change the motion of an object. The metric unit of force is the Newton, defined as the force required to accelerate a standard 1-kg mass from rest to a speed of 1 m/s in 1 s.

Scientific notation means, for example, writing 3.2×10^5 rather than 320000.

Writing numbers with the correct number of significant figures correctly communicates how accurate they are. As a rule of thumb, the final result of a calculation is no more accurate than, and should have no more significant figures than, the least accurate piece of data.

Nature behaves differently on large and small scales. Galileo showed that this results fundamentally from the way area and volume scale. Area scales as the second power of length, $A \propto L^2$, while volume scales as length to the third power, $V \propto L^3$.

An order of magnitude estimate is one in which we do not attempt or expect an exact answer. The main reason why the uninitiated have trouble with order-of-magnitude estimates is that the human brain does not intuitively make accurate estimates of area and volume. Estimates of area and volume should be approached by first estimating linear dimensions, which one's brain has a feel for.

Velocity, $\mathrm{d}x/\,\mathrm{d}t$, measures how fast an object is moving. Acceleration, $\mathrm{d}^2x/\,\mathrm{d}t^2$, measures how quickly its velocity is changing. For motion with constant acceleration, we have these useful relations:

$$a = \frac{\Delta v}{\Delta t}$$
$$x = \frac{1}{2}at^2 + v_o t + x_o$$
$$v_f^2 = v_o^2 + 2a\Delta x$$

Chapter 1, Conservation of Mass, page 55

Conservation laws are the foundation of physics. A conservation law states that a certain quantity can be neither created nor destroyed; the total amount of it remains the same.

Mass is a conserved quantity in classical physics, i.e. physics before Einstein. This is plausible, since we know that matter is composed of subatomic particles; if the particles are neither created or destroyed, then it makes sense that the total mass will remain the same. There are two ways of defining mass.

Gravitational mass is defined by measuring the effect of gravity on a particular object, and comparing with some standard object, taking care to test both objects at a location where the strength of gravity is the same.

Inertial mass is defined by measuring how much a particular object resists a change in its state of motion. For instance, an object placed on the end of a spring will oscillate if the spring is initially compressed, and a more massive object will take longer to complete one oscillation.

Inertial and gravitational mass are equivalent: experiments show to a very high degree of precision that any two objects with the same inertial mass have the same gravitational mass as well.

The definition of inertial mass depends on a correct but counterintuitive assumption: that an object resists a change in its state of motion. Most people intuitively believe that motion has a natural tendency to slow down. This cannot be correct as a general statement, because "to slow down" is not a well-defined concept unless we specify what we are measuring motion relative to. This insight is credited to Galileo, and the general principle of *Galilean relativity* states that the laws of physics are the same in all inertial frames of reference. In other words, there is no way to distinguish a moving frame of reference from one that is at rest. To establish which frames of reference are inertial, we first must find one inertial frame in which objects appear to obey Galilean relativity. The surface of the earth is an inertial frame to a reasonably good approximation, and the frame of reference of the stars is an even better one. Once we have found one inertial frame of reference, any other frame is inertial which is moving in a straight line at constant velocity relative to the first one. For instance, if the surface of the earth is an approximately inertial frame, then a train traveling in a straight line at constant speed is also approximately an inertial frame.

The unit of mass is the kilogram, which, along with the meter and the second, forms the basis for the SI system of units (also known as the mks system). A fundamental skill in science is to know the definitions of the most common metric prefixes, which are summarized on page 967, and to be able to convert among them.

One consequence of Einstein's theory of special relativity is that *mass can be converted to energy and energy to mass.* This prediction has been verified amply by experiment. Thus the conserved quantity is not really mass but rather the total "mass-energy," $m + E/c^2$, where c is the speed of light. Since the speed of light is a large number, the E/c^2 term is ordinarily small in everyday life, which is why we can usually neglect it.

Chapter 2, Conservation of Energy, page 73

We observe that certain processes are physically impossible. For example, there is no process that can heat up an object without using up fuel or having some other side effect such as cooling a different object. We find that we can neatly separate the possible processes from the impossible by defining a single numerical quantity, called *energy*, which is conserved. Energy comes in many forms, such as heat, motion, sound, light, the energy required to melt a solid, and gravitational energy (e.g. the energy that depends on the distance between a rock and the earth). Because it has so many forms, we can arbitrarily choose one form, heat, in order to define a standard unit for our numerical scale of energy. Energy is measured in units of joules (J), and one joule can be defined as the amount of energy required in order to raise the termperature of a certain amount of water by a certain number of degrees. (The numbers are not worth memorizing.) *Power* is defined as the rate of change of energy $P = \mathrm{d}E/\,\mathrm{d}t$, and the unit of power is the watt, $1\ \mathrm{W} = 1\ \mathrm{J/s}$.

Once we have defined one type of energy numerically, we can perform experiments that establish the mathematical rules governing other types of energy. For example, in his paddlewheel experiment, James Joule allowed weights to drop through a certain height and spin paddlewheels inside sealed canisters of water, thereby heating the water through friction. Since in this book we define the joule unit in terms of the temperature of water, we can think of the paddlewheel experiment as establishing a rule for the *gravitational energy* of a mass which is at a certain height,

$$\mathrm{d}U_g = mg\,\mathrm{d}y,$$

where $\mathrm{d}U_g$ is the infinitesimal change in the gravitational energy of a mass m when its height is changed by an infinitesimal amount $\mathrm{d}y$ in the vertical direction. The quantity g is called the *gravitational field*, and at the earth's surface it has a numerical value of about $10\ \mathrm{J/kg{\cdot}m}$. That is, about 10 joules of energy are required in order to raise a one-kilogram mass by one meter. (The gravitational field g also has the interpretation that when we drop an object, its acceleration, $\mathrm{d}^2y/\,\mathrm{d}t^2$, is equal to g.)

Using similar techniques, we find that the energy of a moving object, called its *kinetic energy*, is given by

$$K = \frac{1}{2}mv^2,$$

where m is its mass and v its velocity. The proportionality factor equals $1/2$ exactly by the design of the SI system of units, and since the SI is based on the meter, the kilogram, and the second, the joule is considered to be a derived unit, $1\ \mathrm{J} = 1\ \mathrm{kg{\cdot}m^2/s^2}$.

When the interaction energy U has a local maximum or minimum with respect to the position of an object ($\mathrm{d}U/\,\mathrm{d}x = 0$), then the object is in *equilibrium* at that position. For example, if a weight is hanging from a rope, and is initially at rest at the bottom, then it must remain at rest, because this is a position of minimum gravitational energy U_g; to move, it would have to increase both its kinetic and its gravitational energy, which would violate conservation of energy, since the total energy would increase.

Since kinetic energy is independent of the direction of motion, conservation of energy is often insufficient to predict the direction of an object's motion. However, many of the physically impossible motions can be ruled out by the trick of imposing conservation of energy in some other frame of reference. By this device, we can solve the important problem of *projectile motion:* even if the projectile has horizontal motion, we can imagine ourselves in a frame of reference in which we are moving along with the projectile horizontally. In this frame of reference, the projectile has no horizontal motion, and its vertical motion has constant acceleration g. Switching back to a frame of reference in which its horizontal velocity is not zero, we find that a projectile's horizontal and vertical motions are independent, and that the horizontal motion is at constant velocity.

Even in one-dimensional motion, it is seldom possible to solve real-world problems and predict the motion of an object in closed form. However, there are straightforward numerical techniques for solving such problems.

From observations of the motion of the planets, we infer that the *gravitational interaction between any two objects* is given by $U_g = -Gm_1m_2/r$, where r is the distance between them. When the sizes of the objects are not small compared to their separation, the definition of r becomes vague; for this reason, we should interpret this fundamentally as the law governing the gravitational interactions between individual atoms. However, in the special case of a spherically symmetric mass distribution, there is a shortcut: the *shell theorem* states that the gravitational interaction between a spherically symmetric shell of mass and a particle on the outside of the shell is the same as if the shell's mass had all been concentrated at its center. An astronomical body like the earth can be broken down into concentric shells of mass, and so its gravitational interactions with external objects can also be calculated simply by using the center-to-center distance.

Energy appears to come in a bewildering variety of forms, but matter is made of atoms, and thus if we restrict ourselves to the study of mechanical systems (containing material objects, not light), all the forms of energy we observe must be explainable in terms of the behabior and interactions of atoms. Indeed, at the atomic level the picture is much simpler. Fundamentally, all the familiar forms of mechanical energy arise from either the kinetic energy of atoms or the energy they have because they interact with each other via gravitational or electrical forces. For example, when we stretch a spring, we distort the latticework of atoms in the metal, and this change in the interatomic distances involves an increase in the atoms' electrical energies.

An equilibrium is a local minimum of $U(x)$, and up close, any minimum looks like a parabola. Therefore, small oscillations around an equilibrium exhibit universal behavior, which depends only on the object's mass, m, and on the tightness of curvature of the minimum, parametrized by the quantity $k = \mathrm{d}^2U/\mathrm{d}x^2$. The oscillations are sinusoidal as a function of time, and the period is $T = 2\pi\sqrt{m/k}$, independent of amplitude. When oscillations are small enough for these statements to be good approximations, we refer to them the oscillations as *simple harmonic*.

Chapter 3, Conservation of Momentum, page 129

Since the kinetic energy of a material object depends on v^2, it isn't obvious that conservation of energy is consistent with Galilean relativity. Even if a certain mechanical system conserves energy in one frame of reference, the velocities involved will be different as measured in another frame, and therefore so will the kinetic energies. It turns out that consistency is achieved only if there is a new conservation law, conservation of *momentum*,

$$\mathbf{p} = m\mathbf{v}.$$

In one dimension, the direction of motion is described using positive and negative signs of the velocity $\mathbf{v}$, and since mass is always positive, the momentum carries the same sign. Thus conservation of momentum, unlike conservation of energy, makes direct predictions about the direction of motion. Although this line of argument was based on the assumption of a mechanical system, momentum need not be mechanical. Light has momentum.

A moving object's momentum equals the sum of the momenta of all its atoms. To avoid having to carry out this sum, we can use the concept of the *center of mass*. The center of mass can be defined as a kind of weighted average of the positions of all the atoms in the object,

$$\mathbf{x}_{cm} = \frac{\sum m_j \mathbf{x}_j}{\sum m_j},$$

and although the definition does involve a sum, we can often locate the center of mass by symmetry or by physically determining an object's balance point. The total momentum of the object is then given by

$$\mathbf{p}_{total} = m_{total}\mathbf{v}_{cm}.$$

The rate of transfer of momentum is called *force*, $\mathbf{F} = \mathrm{d}\mathbf{p}/\,\mathrm{d}t$, and is measured in units of newtons, 1 N = 1 kg·m/s^2. As a direct consequence of conservation of momentum, we have the following statements, known as *Newton's laws of motion:*

> If the total force on an object is zero, it remains in the same state of motion.
>
> $\mathbf{F} = \mathrm{d}\mathbf{p}/\,\mathrm{d}t$
>
> Forces always come in pairs: if object A exerts a force on object B, then object B exerts a force on object A which is the same strength, but in the opposite direction.

Although the fundamental forces at the atomic level are gravity, electromagnetism, and nuclear forces, we use a different and more practical classification scheme in everyday situations. In this scheme, the forces between solid objects are described as follows:

A normal force, F_n,	is perpendicular to the surface of contact, and prevents objects from passing through each other by becoming as strong as necessary (up to the point where the objects break). "Normal" means perpendicular.
Static friction, F_s,	is parallel to the surface of contact, and prevents the surfaces from starting to slip by becoming as strong as necessary, up to a maximum value of $F_{s,max}$. "Static" means not moving, i.e. not slipping.
Kinetic friction, F_k,	is parallel to the surface of contact, and tends to slow down any slippage once it starts. "Kinetic" means moving, i.e. slipping.

Work is defined as the transfer of energy by a force. ("By a force" is meant to exclude energy transfer by heat conduction.) The *work theorem* states that when a force occurs at a single point of contact, the amount of energy transferred by that force is given by $\mathrm{d}W = \mathbf{F}\cdot\mathrm{d}\mathbf{x}$, where $\mathrm{d}\mathbf{x}$ is the distance traveled by the point of contact. The *kinetic energy theorem* is $\mathrm{d}K_{cm} = \mathbf{F}_{total}\cdot\mathrm{d}\mathbf{x}_{cm}$, where $\mathrm{d}K_{cm}$ is the change in the energy, $(1/2)mv_{cm}^2$, an object possesses due to the motion of its center of mass, $\mathbf{F}_{total}$ is the total force acting on the object, and $\mathrm{d}\mathbf{x}_{cm}$ is the distance traveled by the center of mass.

The relationship between force and interaction energy is $U = -\,\mathrm{d}F/\,\mathrm{d}x$. Any interaction can be described either by giving the force as a function of distance or the interaction energy as a function of distance; the other quantity can then be found by integration or differentiation.

An oscillator subject to friction will, if left to itself, suffer a gradual decrease in the amplitude of its motion as mechanical energy is transformed into heat. The *quality factor*, Q, is defined as the number of oscillations required for the mechanical energy to fall off by a factor of $e^{2\pi} \approx 535$. To maintain an oscillation indefinitely, an external force must do work to replace this energy. We assume for mathematical simplicity that the external force varies sinusoidally with time, $F = F_m \sin \omega t$. If this force is applied for a long time, the motion approaches a steady state, in which the oscillator's motion is sinusoidal, matching the driving force in frequency but not in phase. The amplitude of this steady-state motion, A, exhibits the phenomenon of *resonance:* the amplitude is maximized at a driving frequency which, for large Q, is essentially the same as the natural frequency of the free vibrations, ω_f (and for large Q this is also nearly the same as $\omega_o = \sqrt{k/m}$). When the energy of the steady-state oscillations is graphed as a function of frequency, both the height and the width of the resonance peak depend on Q. The peak is taller for greater Q, and its full width at half-maximum is $\Delta\omega \approx \omega_o/Q$. For small values of Q, all these approximations become worse, and at $Q < 1/2$ qualitatively different behavior sets in.

For three-dimensional motion, a moving object's motion can be described by three different velocities, $v_x = \mathrm{d}x/\,\mathrm{d}t$, and similarly for v_y and v_z. Thus conservation of momentum becomes three different conservation laws: conservation of $p_x = mv_x$, and so on. The principle of *rotational invariance* says that the laws of physics are the same regardless of how we change the orientation of our laboratory: there is no preferred direction in space. As a consequence of this, no matter how we choose our x, y, and z coordinate axes, we will still have conservation of p_x, p_y, and p_z. To simplify notation, we define a momentum *vector*, $\mathbf{p}$, which is a single symbol that stands for all the momentum information contained in the components p_x, p_y, and p_z. The concept of a vector is more general than its application to the momentum: any quantity that has a direction in space is considered a vector, as opposed to a *scalar* like time or temperature. The following table summarizes some vector operations.

operation	*definition*
$\lvert\mathbf{vector}\rvert$	$\sqrt{vector_x^2 + vector_y^2 + vector_z^2}$
$\mathbf{vector} + \mathbf{vector}$	Add component by component.
$\mathbf{vector} - \mathbf{vector}$	Subtract component by component.
$\mathbf{vector} \cdot scalar$	Multiply each component by the scalar.
$\mathbf{vector}\ /\ scalar$	Divide each component by the scalar.

Differentiation and integration of vectors is defined component by component.

There is only one meaningful (rotationally invariant) way of defining a multiplication of vectors whose result is a scalar, and it is known as the vector *dot product:*

$$\begin{aligned}\mathbf{b}\cdot\mathbf{c} &= b_x c_x + b_y c_y + b_z c_z \\ &= |\mathbf{b}|\,|\mathbf{c}|\,\cos\theta_{bc}.\end{aligned}$$

The dot product has most of the usual properties associated with multiplication, except that there is no "dot division."

Chapter 4, Conservation of Angular Momentum, page 245

Angular momentum is a conserved quantity. For motion confined to a plane, the angular momentum of a material particle is

$$L = mv_\perp r,$$

where r is the particle's distance from the point chosen as the axis, and $v_\perp$ is the component of its velocity vector that is perpendicular to the line connecting the particle to the axis. The choice of axis is arbitrary. In a plane, only two directions of rotation are possible, clockwise and counterclockwise. One of these is considered negative and the other positive. Geometrically, angular momentum is related to rate at which area is swept out by the line segment connecting the particle to the axis.

Torque is the rate of change of angular momentum, $\tau = \mathrm{d}L/\,\mathrm{d}t$. The torque created by a given force can be calculated using any of the relations

$$\begin{aligned}\tau &= rF\sin\theta_{rF}\\ &= rF_\perp\\ &= r_\perp F,\end{aligned}$$

where the subscript $\perp$ indicates a component perpendicular to the line connecting the axis to the point of application of the force.

In the special case of a *rigid body* rotating in a single plane, we define

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \qquad \text{[angular velocity]}$$

and

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}, \qquad \text{[angular acceleration]}$$

in terms of which we have

$$L = I\omega$$

and

$$\tau = I\alpha,$$

where the *moment of inertia*, I, is defined as

$$I = \sum m_i r_i^2,$$

summing over all the atoms in the object (or using calculus to perform a continuous sum, i.e. an integral). The relationship between the angular quantities and the linear ones is

$$\begin{aligned}
v_t &= \omega r && \text{[tangential velocity of a point]}\\
v_r &= 0 && \text{[radial velocity of a point]}\\
a_t &= \alpha r. && \text{[radial acceleration of a point at a distance } r \text{ from the axis]}\\
a_r &= \omega^2 r && \text{[radial acceleration of a point at a distance } r \text{ from the axis]}
\end{aligned}$$

In three dimensions, torque and angular momentum are vectors, and are expressed in terms of the vector *cross product*, which is the only rotationally invariant way of defining a multiplication of two vectors that produces a third vector:

$$\begin{aligned}\mathbf{L} &= \mathbf{r}\times\mathbf{p}\\ \boldsymbol{\tau} &= \mathbf{r}\times\mathbf{F}\end{aligned}$$

In general, the cross product of vectors $\mathbf{b}$ and $\mathbf{c}$ has magnitude

$$|\mathbf{b} \times \mathbf{c}| = |\mathbf{b}|\,|\mathbf{c}|\,\sin\theta_{bc},$$

which can be interpreted geometrically as the area of the parallelogram formed by the two vectors when they are placed tail-to-tail. The direction of the cross product lies along the line which is perpendicular to both vectors; of the two such directions, we choose the one that is right-handed, in the sense that if we point the fingers of the flattened right hand along $\mathbf{b}$, then bend the knuckles to point the fingers along $\mathbf{c}$, the thumb gives the direction of $\mathbf{b} \times \mathbf{c}$. In terms of components, the cross product is

$$\begin{aligned}(\mathbf{b} \times \mathbf{c})_x &= b_y c_z - c_y b_z \\ (\mathbf{b} \times \mathbf{c})_y &= b_z c_x - c_z b_x \\ (\mathbf{b} \times \mathbf{c})_z &= b_x c_y - c_x b_y\end{aligned}$$

The cross product has the disconcerting properties

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a} \qquad \text{[noncommutative]}$$

and

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \neq (\mathbf{a} \times \mathbf{b}) \times \mathbf{c} \qquad \text{[nonassociative]},$$

and there is no "cross-division."

For rigid-body rotation in three dimensions, we define an angular velocity vector $\boldsymbol{\omega}$, which lies along the axis of rotation and bears a right-hand relationship to it. Except in special cases, there is no scalar moment of inertia for which $\mathbf{L} = I\boldsymbol{\omega}$; the moment of inertia must be expressed as a matrix.

Chapter 5, Thermodynamics, page 299

A fluid is any gas or liquid, but not a solid; fluids do not exhibit shear forces. A fluid in equilibrium exerts a force on any surface which is proportional to the surface's area and perpendicular to the surface. We can therefore define a quantity called the *pressure*, P, which is ratio of force to area,

$$P = \frac{F_\perp}{A},$$

where the subscript $\perp$ indicates the component of the fluid's force which is perpendicular to the surface.

Usually it is only the difference in pressure between the two sides of a surface that is physically significant. Pressure doesn't just "press down" on things; air pressure upward under your chin is the same as air pressure downward on your shoulders. In a fluid acted on by gravity, pressure varies with depth according to the equation

$$\mathrm{d}P = -\rho\mathbf{g} \cdot \mathrm{d}\mathbf{y}.$$

This equation is only valid if the fluid is in equilibrium, and if g and r are constant with respect to height.

Temperature can be defined according to the volume of an ideal gas under conditions of standard pressure. The Kelvin scale of temperature used throughout this book equals zero at absolute zero, the temperature at which all random molecular motion ceases, and equals 273 K at the freezing point of water. We can get away with using the Celsius scale as long as we are only interested in temperature differences; a difference of 1 degree C is the same as a difference of 1 degree K.

It is an observed fact that ideal gases obey the *ideal gas law*,

$$PV = nkT,$$

and this equation can be explained by the kinetic theory of heat, which states that the gas experts pressure on its container because its molecules are constantly in motion. In the kinetic theory of heat, the temperature of a gas is proportional to the average energy per molecule.

Not all the heat energy in an object can be extracted to do mechanical work. We therefore describe heat as a lower grade of energy than other forms of energy. Entropy is a measure of how much of a system's energy is inaccessible to being extracted, even by the most efficient heat engine; a high entropy corresponds to a low grade of energy. The change in a system's entropy when heat Q is deposited into it is

$$\Delta S = \frac{Q}{T}.$$

The efficiency of any heat engine is defined as

$$\text{efficiency} = \frac{\text{energy we get in useful form}}{\text{energy we pay for}},$$

and the efficiency of a Carnot engine, the most efficient of all, is

$$\text{efficiency} = 1 - \frac{T_L}{T_H}.$$

These results are all closely related. For instance, example 11 on page 315 uses $\Delta S = Q/T$ and efficiency $= 1 - T_L/T_H$ to show that a Carnot engine doesn't change the entropy of the universe.

Fundamentally, entropy is defined as the being proportional to the natural logarithm of the number of states available to a system, and the above equation then serves as a definition of temperature. The entropy of a closed system always increases; this is the second law of thermodynamics.

Chapter 6, Waves, page 343

Wave motion differs in three important ways from the motion of material objects:

Waves obey the principle of superposition. When two waves collide, they simply add together.

The medium is not transported along with the wave. The motion of any given point in the medium is a vibration about its equilibrium location, not a steady forward motion.

The velocity of a wave depends on the medium, not on the amount of energy in the wave. (For some types of waves, notably water waves, the velocity may also depend on the shape of the wave.)

Sound waves consist of increases and decreases (typically very small ones) in the density of the air. Light is a wave, but it is a vibration of electric and magnetic fields, not of any physical medium. Light can travel through a vacuum.

A periodic wave is one that creates a periodic motion in a receiver as it passes it. Such a wave has a well-defined period and frequency, and it will also have a wavelength, which is the distance in space between repetitions of the wave pattern. The velocity, frequency, and wavelength of a periodic wave are related by the equation

$$v = f\lambda.$$

A wave emitted by a moving source will undergo a *Doppler shift* in wavelength and frequency. The shifted wavelength is given by the equation

$$\lambda' = \left(1 - \frac{u}{v}\right)\lambda,$$

where v is the velocity of the waves and u is the velocity of the source, taken to be positive or negative so as to produce a Doppler-lengthened wavelength if the source is receding and a Doppler-shortened one if it approaches. A similar shift occurs if the observer is moving, and in general the Doppler shift depends approximately only on the relative motion of the source and observer if their velocities are both small compared to the waves' velocity. (This is not just approximately but exactly true for light waves, as required by Einstein's theory of relativity.)

Whenever a wave encounters the boundary between two media in which its speeds are different, part of the wave is reflected and part is transmitted. The reflection is always reversed front-to-back, but may also be inverted in amplitude. Whether the reflection is inverted depends on how the wave speeds in the two media compare, e.g. a wave on a string is uninverted when it is reflected back into a segment of string where its speed is lower. The greater the difference in wave speed between the two media, the greater the fraction of the wave energy that is reflected. Surprisingly, a wave in a dense material like wood will be strongly reflected back into the wood at a wood-air boundary.

A one-dimensional wave confined by highly reflective boundaries on two sides will display motion which is periodic. For example, if both reflections are inverting, the wave will have a period equal to twice the time required to traverse the region, or to that time divided by an integer. An important special case is a sinusoidal wave; in this case, the wave forms a stationary pattern composed of a superposition of sine waves moving in opposite direction.

Chapter 7, Relativity, page 385

Experiments show that space and time do not have the properties claimed by Galileo and Newton. Time and space as seen by one observer are distorted compared to another observer's perceptions if they are moving relative to each other. This distortion is quantified by the factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where v is the relative velocity of the two observers, and c is a universal velocity that is the same in all frames of reference. Light travels at c. A clock appears to run fastest to an observer who is not in motion relative to it, and appears to run too slowly by a factor of γ to an observer who has a velocity v relative to the clock. Similarly, a meter-stick appears longest to an observer who sees it at rest, and appears shorter to other observers. Time and space are relative, not absolute. In particular, there is no well-defined concept of simultaneity.

All of these strange effects, however, are very small when the relative velocities are small compared to c. This makes sense, because Newton's laws have already been thoroughly tested by experiments at such speeds, so a new theory like relativity must agree with the old one in their realm of common applicability. This requirement of backwards-compatibility is known as the correspondence principle.

Relativity has implications not just for time and space but also for the objects that inhabit time and space. The correct relativistic equation for momentum is

$$p = m\gamma v,$$

which is similar to the classical $p = mv$ at low velocities, where $\gamma \approx 1$, but diverges from it more and more at velocities that approach c. Since γ becomes infinite at $v = c$, an infinite force would be required in order to give a material object enough momentum to move at the speed of light. In other words, material objects can only move at speeds lower than c. Relativistically, mass and energy are not separately conserved. Mass and energy are two aspects of the same phenomenon, known as mass-energy, and they can be converted to one another according to the equation

$$E = mc^2.$$

The mass-energy of a moving object is $\mathcal{E} = m\gamma c^2$. When an object is at rest, $\gamma = 1$, and the mass-energy is simply the energy-equivalent of its mass, mc^2. When an object is in motion, the excess mass-energy, in addition to the mc^2, can be interpreted as its kinetic energy.

Chapter 8, Atoms and Electromagnetism, page 459

All the forces we encounter in everyday life boil down to two basic types: gravitational forces and electrical forces. A force such as friction or a "sticky force" arises from electrical forces between individual atoms.

Just as we use the word mass to describe how strongly an object participates in gravitational forces, we use the word *charge* for the intensity of its electrical forces. There are two types of charge. Two charges of the same type repel each other, but objects whose charges are different attract each other. Charge is measured in units of coulombs (C).

Mobile charged particle model: A great many phenomena are easily understood if we imagine matter as containing two types of charged particles, which are at least partially able to move around.

Positive and negative charge: Ordinary objects that have not been specially prepared have both types of charge spread evenly throughout them in equal amounts. The object will then tend not to exert electrical forces on any other object, since any attraction due to one type of charge will be balanced by an equal repulsion from the other. (We say "tend not to" because bringing the object near an object with unbalanced amounts of charge could cause its charges to separate from each other, and the force would no longer cancel due to the unequal distances.) It therefore makes sense to describe the two types of charge using positive and negative signs, so that an unprepared object will have zero *total* charge.

The *Coulomb force law* states that the magnitude of the electrical force between two charged particles is given by

$$|F| = \frac{k|q_1||q_2|}{r^2}.$$

Conservation of charge: An even more fundamental reason for using positive and negative signs for charge is that with this definition the total charge of a closed system is a conserved quantity.

Quantization of charge: Millikan's oil drop experiment showed that the total charge of an object could only be an integer multiple of a basic unit of charge, e. This supported the idea the the "flow" of electrical charge was the motion of tiny particles rather than the motion of some sort of mysterious electrical fluid.

Einstein's analysis of Brownian motion was the first definitive proof of the existence of atoms. Thomson's experiments with vacuum tubes demonstrated the existence of a new type of microscopic particle with a very small ratio of mass to charge. Thomson correctly interpreted these as building blocks of matter even smaller than atoms: the first discovery of subatomic particles. These particles are called electrons.

The above experimental evidence led to the first useful model of the interior structure of atoms, called the raisin cookie model. In the raisin cookie model, an atom consists of a relatively large, massive, positively charged sphere with a certain number of negatively charged electrons embedded in it.

Rutherford and Marsden observed that some alpha particles from a beam striking a thin gold foil came back at angles up to 180 degrees. This could not be explained in the then-favored raisin-cookie model of the atom, and led to the adoption of the planetary model of the atom, in which the electrons orbit a tiny, positively-charged nucleus. Further experiments showed that the nucleus itself was a cluster of positively-charged protons and uncharged neutrons.

Radioactive nuclei are those that can release energy. The most common types of radioactivity are alpha decay (the emission of a helium nucleus), beta decay (the transformation of a neutron into a proton or vice-versa), and gamma decay (the emission of a type of very-high-frequency light). Stars are powered by nuclear fusion reactions, in which two light nuclei collide and form a bigger nucleus, with the release of energy.

Human exposure to ionizing radiation is measured in units of millirem. The typical person is exposed to about 100 mrem worth of natural background radiation per year.

Chapter 9, DC Circuits, page 515

All electrical phenomena are alike in that that arise from the presence or motion of charge. Most practical electrical devices are based on the motion of charge around a complete circuit, so that the charge can be recycled and does not hit any dead ends. The most useful measure of the flow of charge is *current*,

$$I = \frac{\mathrm{d}q}{\mathrm{d}t}.$$

An electrical device whose job is to transform energy from one form into another, e.g. a lightbulb, uses power at a rate which depends both on how rapidly charge is flowing through it and on how much work is done on each unit of charge. The latter quantity is known as the voltage difference between the point where the current enters the device and the point where the current leaves it. Since there is a type of electrical energy associated with electrical forces, the amount of work they do is equal to the difference in potential energy between the two points, and we therefore define voltage differences directly in terms of electrical energy,

$$\Delta V = \frac{\Delta U_{elec}}{q}.$$

The rate of power dissipation is

$$P = I\Delta V.$$

Many important electrical phenomena can only be explained if we understand the mechanisms of current flow at the atomic level. In metals, currents are carried by electrons, in liquids by ions. Gases are normally poor conductors unless their atoms are subjected to such intense electrical forces that the atoms become ionized.

Many substances, including all solids, respond to electrical forces in such a way that the flow of current between two points is proportional to the voltage difference between those points (assuming the voltage difference is small). Such a substance is called ohmic, and an object made out of an ohmic substance can be rated in terms of its resistance,

$$R = \frac{\Delta V}{I}$$

An important corollary is that a perfect conductor, with $R = 0$, must have constant voltage everywhere within it.

A schematic is a drawing of a circuit that standardizes and stylizes its features to make it easier to understand. Any circuit can be broken down into smaller parts. For instance, one big circuit may be understood as two small circuits in series, another as three circuits in parallel. When circuit elements are combined in parallel and in series, we have two basic rules to guide us in understanding how the parts function as a whole:

> *The junction rule:* In any circuit that is not storing or releasing charge, conservation of charge implies that the total current flowing out of any junction must be the same as the total flowing in.
>
> *The loop rule:* Assuming the standard convention for plus and minus signs, the sum of the voltage drops around any closed loop in a circuit must be zero.

The simplest application of these rules is to pairs of resistors combined in series or parallel. In such cases, the pair of resistors acts just like a single unit with a certain resistance value, called their equivalent resistance. Resistances in series add to produce a larger equivalent resistance,

$$R = R_1 + R_2,$$

because the current has to fight its way through both resistances. Parallel resistors combine to produce an equivalent resistance that is smaller than either individual resistance,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2},$$

because the current has two different paths open to it.

An important example of resistances in parallel and series is the use of voltmeters and ammeters in resistive circuits. A voltmeter acts as a large resistance in parallel with the resistor across which the voltage drop is being measured. The fact that its resistance is not infinite means that it alters the circuit it is being used to investigate, producing a lower equivalent resistance. An ammeter acts as a small resistance in series with the circuit through which the current is to be determined. Its resistance is not quite zero, which leads to an increase in the resistance of the circuit being tested.

Chapter 10, Fields, page 563

Newton conceived of a universe where forces reached across space instantaneously, but we now know that there is a delay in time before a change in the configuration of mass and charge in one corner of the universe will make itself felt as a change in the forces experienced far away. We imagine the outward spread of such a change as a ripple in an invisible universe-filling *field of force.*

As an alternative to our earlier energy-based definition, we can define the *gravitational field* at a given point as the force per unit mass exerted on objects inserted at that point, and likewise the *electric field* is defined as the force per unit charge. These fields are vectors, and the fields generated by multiple sources add according to the rules of vector addition.

The *relationship between the electric field and the voltage* is

$$\begin{aligned}\frac{\partial V}{\partial x} &= -E_x \\ \frac{\partial V}{\partial y} &= -E_y \\ \frac{\partial V}{\partial z} &= -E_z,\end{aligned}$$

which can be notated more compactly as a gradient,

$$\mathbf{E} = -\nabla V.$$

Fields of force contain energy, and the density of energy is proportional to the square of the magnitude of the field,

$$\begin{aligned}\mathrm{d}U_g &= -\frac{1}{8\pi G}g^2\,\mathrm{d}v \\ \mathrm{d}U_e &= \frac{1}{8\pi k}E^2\,\mathrm{d}v \\ \mathrm{d}U_m &\propto B^2\,\mathrm{d}v\end{aligned}$$

The equation for the energy stored in the magnetic field is given explicitly in the next chapter; for now, we only need the fact that it behaves in the same general way as the first two equations: the energy density is proportional to the square of the field. In the case of static electric fields, we can calculate potential energy either using the previous definition in terms of mechanical work or by calculating the energy stored in the fields. If the fields are not static, the old method gives incorrect results and the new one must be used.

Capacitance, C, and inductance, L, are defined as

$$U_C = \frac{1}{2C}q^2$$

and

$$U_L = \frac{L}{2}I^2,$$

measured in units of farads and henries, respectively. The voltage across a capacitor or inductor is given by

$$V_C = \frac{q}{C}$$

or

$$|V_L| = \left| L \frac{\mathrm{d}I}{\mathrm{d}t} \right|.$$

In the equation for the inductor, the direction of the voltage drop (plus or minus sign) is such that the inductor resists the change in current. Although the equation for the voltage across an inductor follows directly from fundamental arguments concerning the energy stored in the magnetic field, the result is a surprise: the voltage drop implies the existence of electric fields which are not created by charges. This is an *induced electric field*, discussed in more detail in the next chapter.

A series LRC circuit exhibits *oscillation*, and, if driven by an external voltage, resonates. The Q of the circuit relates to the resistance value. For large Q, the resonant frequency is

$$\omega \approx \frac{1}{\sqrt{LC}}.$$

A series RC or RL circuit exhibits exponential *decay*,

$$q = q_o \exp\left(-\frac{t}{RC}\right)$$

or

$$I = I_o \exp\left(-\frac{R}{L}t\right),$$

and the quantity RC or L/R is known as the time constant.

When driven by a sinusoidal AC voltage with amplitude $\tilde{V}$, a capacitor, resistor, or inductor responds with a current having amplitude

$$\tilde{I} = \frac{\tilde{V}}{Z},$$

where the *impedance*, Z, is a frequency-dependent quantity having units of ohms. In a capacitor, the current has a phase that is 90° ahead of the voltage, while in an inductor the current is 90° behind. We can represent these phase relationships by defining the impedances as complex numbers:

$$Z_C = -\frac{i}{\omega C}$$
$$Z_R = R$$
$$Z_L = i\omega L$$

The arguments of the complex impedances are to be interpreted as phase relationships between the oscillating voltages and currents. The complex impedances defined in this way combine in series and parallel according to the same rules as resistances.

When a voltage source is driving a load through a transmission line, the maximum power is delivered to the load when the impedances of the line and the load are matched.

Gauss' law states that for any region of space, the flux through the surface,

$$\Phi = \sum \mathbf{E}_j \cdot \mathbf{A}_j,$$

is related by

$$\Phi = 4\pi k q_{in}$$

to the charge enclosed within the surface.

Chapter 11, Electromagnetism, page 653

Relativity implies that there must be an interaction between moving charges and other moving charges. This *magnetic* interaction is in addition to the usual electrical one. The magnetic field can be defined in terms of the magnetic force exerted on a test charge,

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B},$$

or, alternatively, in terms of the torque on a magnetic test dipole,

$$|B| = \frac{\tau}{|\mathbf{m}_t| \sin\theta},$$

where θ is the angle between the dipole vector and the field. The magnetic dipole moment $\mathbf{m}$ of a loop of current has magnitude $m = IA$, and is in the (right-handed) direction perpendicular to the loop.

The magnetic field has no sources or sinks. Gauss' law for magnetism is

$$\Phi_B = 0.$$

The external magnetic field of a long, straight wire is

$$B = \frac{2kI}{c^2 R},$$

forming a right-handed circular pattern around the wire.

The energy of the magnetic field is

$$\mathrm{d}U_m = \frac{c^2}{8\pi k} B^2 \, \mathrm{d}v.$$

The magnetic field resulting from a set of currents can be computed by finding a set of dipoles that combine to give those currents. The field of a dipole is

$$B_z = \frac{km}{c^2}\left(3\cos^2\theta - 1\right) r^{-3}$$
$$B_R = \frac{km}{c^2}\left(3\sin\theta\ \cos\theta\right) r^{-3},$$

which reduces to $B_z = km/c^2r^3$ in the plane perpendicular to the dipole moment. By constructing a current loop out of dipoles, one can prove the *Biot-Savart law*,

$$\mathrm{d}\mathbf{B} = \frac{kI\,\mathrm{d}\boldsymbol{\ell} \times \mathbf{r}}{c^2 r^3},$$

which gives the field when we integrate over a closed current loop. All of this is valid only for static magnetic fields.

Ampère's law is another way of relating static magnetic fields to the static currents that created them, and it is more easily extended to nonstatic fields than is the Biot-Savart law. Ampère's law states that the *circulation* of the magnetic field,

$$\Gamma_B = \sum \mathbf{s}_j \cdot \mathbf{B}_j,$$

around the edge of a surface is related to the current $I_{through}$ passing through the surface,

$$\Gamma = \frac{4\pi k}{c^2} I_{through}.$$

In the general nonstatic case, the fundamental laws of physics governing electric and magnetic fields are *Maxwell's equations*, which state that for any closed surface, the fluxes through the surface are

$$\begin{aligned} \Phi_E &= 4\pi k q_{in} \qquad \text{and} \\ \Phi_B &= 0. \end{aligned}$$

For any surface that is not closed, the circulations around the edges of the surface are given by

$$\begin{aligned} \Gamma_E &= -\frac{\partial \Phi_B}{\partial t} \qquad \text{and} \\ c^2\Gamma_B &= \frac{\partial \Phi_E}{\partial t} + 4\pi k I_{through}. \end{aligned}$$

The most important result of Maxwell's equations is the existence of *electromagnetic waves* which propagate at the velocity of light — that's what light is. The waves are transverse, and the electric and magnetic fields are perpendicular to each other. There are no purely electric or purely magnetic waves; their amplitudes are always related to one another by $B = E/c$. They propagate in the right-handed direction given by the cross product $\mathbf{E} \times \mathbf{B}$, and carry momentum $p = U/c$.

A complete statement of Maxwell's equations in the presence of electric and magnetic materials is as follows:

$$\begin{aligned} \Phi_D &= q_{\text{free}} \\ \Phi_B &= 0 \\ \Gamma_E &= -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \\ \Gamma_H &= \frac{\mathrm{d}\Phi_D}{\mathrm{d}t} + I_{\text{free}}, \end{aligned}$$

where the auxiliary fields $\mathbf{D}$ and $\mathbf{H}$ are defined as

$$\begin{aligned} \mathbf{D} &= \epsilon \mathbf{E} \qquad \text{and} \\ \mathbf{H} &= \frac{\mathbf{B}}{\mu}, \end{aligned}$$

and ϵ and μ are the permittivity and permeability of the substance.

Chapter 12, Optics, page 741

The ray model of light: We can understand many phenomena involving light without having to use sophisticated models such as the wave model or the particle model. Instead, we simply describe light according to the path it takes, which we call a ray. The ray model of light is useful when light is interacting with material objects that are much larger than a wavelength of light. Since a wavelength of visible light is so short compared to the human scale of existence, the ray model is useful in many practical cases.

A smooth surface produces specular reflection, in which the reflected ray exits at the same angle with respect to the normal as that of the incoming ray. A rough surface gives diffuse reflection, where a single ray of light is divided up into many weaker reflected rays going in many directions.

Images: A large class of optical devices, including lenses and flat and curved mirrors, operates by bending light rays to form an image. A real image is one for which the rays actually cross at each point of the image. A virtual image, such as the one formed behind a flat mirror, is one for which the rays only appear to have crossed at a point on the image. A real image can be projected onto a screen; a virtual one cannot.

Mirrors and lenses will generally make an image that is either smaller than or larger than the original object. The scaling factor is called the magnification. In many situations, the angular magnification is more important than the actual magnification.

Every lens or mirror has a property called the focal length, which is defined as the distance from the lens or mirror to the image it forms of an object that is infinitely far away. A stronger lens or mirror has a shorter focal length.

Locating images: The relationship between the locations of an object and its image formed by a lens or mirror can always be expressed by equations of the form

$$\begin{aligned}\theta_f &= \pm\theta_i \pm \theta_o \\ \frac{1}{f} &= \pm\frac{1}{d_i} \pm \frac{1}{d_o}.\end{aligned}$$

The choice of plus and minus signs depends on whether we are dealing with a lens or a mirror, whether the lens or mirror is converging or diverging, and whether the image is real or virtual. A method for determining the plus and minus signs is as follows:

1. Use ray diagrams to decide whether θ_i and θ_o vary in the same way or in opposite ways. Based on this, decide whether the two signs in the equation are the same or opposite. If the signs are opposite, go on to step 2 to determine which is positive and which is negative.

2. If the signs are opposite, we need to decide which is the positive one and which is the negative. Since the focal angle is never negative, the smaller angle must be the one with a minus sign.

Once the correct form of the equation has been determined, the magnification can be found via the equation

$$M = \frac{d_i}{d_o}.$$

This equation expresses the idea that the entire image-world is shrunk consistently in all three dimensions.

Refraction: Refraction is a change in direction that occurs when a wave encounters the interface between two media. Together, refraction and reflection account for the basic principles behind nearly all optical devices.

Snell discovered the equation for refraction,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2,$$

[angles measured with respect to the normal]

through experiments with light rays, long before light was proven to be a wave. Snell's law can be proven based on the geometrical behavior of waves. Here n is the index of refraction. Snell invented this quantity to describe the refractive properties of various substances, but it was later found to be related to the speed of light in the substance,

$$n = \frac{c}{v},$$

where c is the speed of light in a vacuum. In general a material's index of refraction is different for different wavelengths of light. Total internal reflection occurs when there is no angle that satisfies Snell's law.

Wave optics: Wave optics is a more general theory of light than ray optics. When light interacts with material objects that are much larger then one wavelength of the light, the ray model of light is approximately correct, but in other cases the wave model is required.

Huygens' principle states that, given a wavefront at one moment in time, the future behavior of the wave can be found by breaking the wavefront up into a large number of small, side-by-side wave peaks, each of which then creates a pattern of circular or spherical ripples. As these sets of ripples add together, the wave evolves and moves through space. Since Huygens' principle is a purely geometrical construction, diffraction effects obey a simple scaling rule: the behavior is unchanged if the wavelength and the dimensions of the diffracting objects are both scaled up or down by the same factor. If we wish to predict the angles at which various features of the diffraction pattern radiate out, scaling requires that these angles depend only on the unitless ratio λ/d, where d is the size of some feature of the diffracting object.

Double-slit diffraction is easily analyzed using Huygens' principle if the slits are narrower than one wavelength. We need only construct two sets of ripples, one spreading out from each slit. The angles of the maxima (brightest points in the bright fringes) and minima (darkest points in the dark fringes) are given by the equation

$$\frac{\lambda}{d} = \frac{\sin\theta}{m},$$

where d is the center-to-center spacing of the slits, and m is an integer at a maximum or an integer plus $1/2$ at a minimum.

If some feature of a diffracting object is repeated, the diffraction fringes remain in the same places, but become narrower with each repetition. By repeating a double-slit pattern hundreds or thousands of times, we obtain a diffraction grating.

A single slit can produce diffraction fringes if it is larger than one wavelength. Many practical instances of diffraction can be interpreted as single-slit diffraction, e.g., diffraction in telescopes. The main thing to realize about single-slit diffraction is that it exhibits the same kind of relationship between λ, d, and angles of fringes as in any other type of diffraction.

Chapter 13, Quantum Physics, page 831

Quantum physics differs from classical physics in many ways, the most dramatic of which is that certain processes at the atomic level, such as radioactive decay, are random rather than deterministic. There is a method to the madness, however: quantum physics still rules out any process that violates conservation laws, and it also offers methods for calculating probabilities numerically. The most important of these generic methods is the law of independent probabilities, which states that if two random events are not related in any way, then the probability that they will both occur equals the product of the two probabilities,

$$\text{probability of A and B} = P_A P_B \qquad \text{[if A and B are independent]}.$$

When discussing a random variable x that can take on a continuous range of values, we cannot assign any finite probability to any particular value. Instead, we define the probability distribution $D(x)$, defined so that its integral over some range of x gives the probability of that range.

In radioactive decay, the time that a radioactive atom has a 50% chance of surviving is called the half-life, $t_{1/2}$. The probability of surviving for two half-lives is $(1/2)(1/2) = 1/4$, and so on. In general, the probability of surviving a time t is given by

$$P_{surv}(t) = 0.5^{t/t_{1/2}}.$$

Related quantities such as the rate of decay and probability distribution for the time of decay are given by the same type of exponential function, but multiplied by certain constant factors.

Around the turn of the twentieth century, experiments began to show problems with the classical wave theory of light. In any experiment sensitive enough to detect very small amounts of light energy, it becomes clear that light energy cannot be divided into chunks smaller than a certain amount. Measurements involving the photoelectric effect demonstrate that this smallest unit of light energy equals hf, where f is the frequency of the light and h is a number known as Planck's constant. We say that light energy is quantized in units of hf, and we interpret this quantization as evidence that light has particle properties as well as wave properties. Particles of light are called photons.

The only method of reconciling the wave and particle natures of light that has stood the test of experiment is the probability interpretation: the probability that the particle is at a given location is proportional to the square of the amplitude of the wave at that location.

One important consequence of wave-particle duality is that we must abandon the concept of the path the particle takes through space. To hold on to this concept, we would have to contradict the well established wave nature of light, since a wave can spread out in every direction simultaneously.

Light is both a particle and a wave. Matter is both a particle and a wave. The equations that connect the particle and wave properties are the same in all cases:

$$E = hf$$
$$p = h/\lambda$$

Unlike the electric and magnetic fields that make up a photon-wave, the electron wavefunction is not directly measurable. Only the square of the wavefunction, which relates to probability, has direct physical significance.

A particle that is bound within a certain region of space is a standing wave in terms of quantum physics. The two equations above can then be applied to the standing wave to yield some important general observations about bound particles:

1. The particle's energy is quantized (can only have certain values).
2. The particle has a minimum energy.
3. The smaller the space in which the particle is confined, the higher its kinetic energy must be.

These immediately resolve the difficulties that classical physics had encountered in explaining observations such as the discrete spectra of atoms, the fact that atoms don't collapse by radiating away their energy, and the formation of chemical bonds.

A standing wave confined to a small space must have a short wavelength, which corresponds to a large momentum in quantum physics. Since a standing wave consists of a superposition of two traveling waves moving in opposite directions, this large momentum should actually be interpreted as an equal mixture of two possible momenta: a large momentum to the left, or a large momentum to the right. Thus it is not possible for a quantum wave-particle to be confined to a small space without making its momentum very uncertain. In general, the Heisenberg uncertainty principle states that it is not possible to know the position and momentum of a particle simultaneously with perfect accuracy. The uncertainties in these two quantities must satisfy the approximate inequality

$$\Delta p \Delta x \gtrsim h.$$

When an electron is subjected to electric forces, its wavelength cannot be constant. The "wavelength" to be used in the equation $p = h/\lambda$ should be thought of as the wavelength of the sine wave that most closely approximates the curvature of the wavefunction at a specific point.

Infinite curvature is not physically possible, so realistic wavefunctions cannot have kinks in them, and cannot just cut off abruptly at the edge of a region where the particle's energy would be insufficient to penetrate according to classical physics. Instead, the wavefunction "tails off" in the classically forbidden region, and as a consequence it is possible for particles to "tunnel" through regions where according to classical physics they should not be able to penetrate. If this quantum tunneling effect did not exist, there would be no fusion reactions to power our sun, because the energies of the nuclei would be insufficient to overcome the electrical repulsion between them.

Hydrogen, with one proton and one electron, is the simplest atom, and more complex atoms can often be analyzed to a reasonably good approximation by assuming their electrons occupy states that have the same structure as the hydrogen atom's. The electron in a hydrogen atom exchanges very little energy or angular momentum with the proton, so its energy and angular momentum are nearly constant, and can be used to classify its states. The energy of a hydrogen state depends only on its n quantum number.

In quantum physics, the angular momentum of a particle moving in a plane is quantized in units of $\hbar$. Atoms are three-dimensional, however, so the question naturally arises of how to deal with angular momentum in three dimensions. In three dimensions, angular momentum is a vector in the direction perpendicular to the plane of motion, such that the motion appears clockwise if viewed along the direction of the vector. Since angular momentum depends on both position and momentum, the Heisenberg uncertainty principle limits the accuracy with which one can know it. The most that can be known about an angular momentum vector is its magnitude and one of its three vector components, both of which are quantized in units of $\hbar$.

In addition to the angular momentum that an electron carries by virtue of its motion through space, it possesses an intrinsic angular momentum with a magnitude of $\hbar/2$. Protons and neutrons also have spins of $\hbar/2$, while the photon has a spin equal to $\hbar$.

Particles with half-integer spin obey the Pauli exclusion principle: only one such particle can exist is a given state, i.e., with a given combination of quantum numbers.

We can enumerate the lowest-energy states of hydrogen as follows:

$n = 1$,	$\ell = 0$,	$\ell_z = 0$,	$s_z = +1/2$ or $-1/2$	two states
$n = 2$,	$\ell = 0$,	$\ell_z = 0$,	$s_z = +1/2$ or $-1/2$	two states
$n = 2$,	$\ell = 1$,	$\ell_z = -1$, 0, or 1,	$s_z = +1/2$ or $-1/2$	six states
...				...

The periodic table can be understood in terms of the filling of these states. The nonreactive noble gases are those atoms in which the electrons are exactly sufficient to fill all the states up to a given n value. The most reactive elements are those with one more electron than a noble gas element, which can release a great deal of energy by giving away their high-energy electron, and those with one electron fewer than a noble gas, which release energy by accepting an electron.

Index