

ball is at rest relative to the earth, she sees both the ball and the earth as moving upward at the same speed. It would be perfectly consistent with conservation of energy if she were to see them maintain this distance from one another for several minutes. In her frame, their kinetic energies would be nonzero, but constant, and the gravitational energy only depends on the separation between the ball and the earth, so it would be constant as well.

Now that we're thinking of the ball and the earth as two objects interacting with one another, it becomes natural to think of them on the same footing. What about the motion of the Earth? The earth feels a gravitational attraction from the ball, just as the ball feels one from the Earth. To make this symmetry more evident, let's imagine two planets of equal mass, Foo and Bar, initially at rest with respect to one another. The Fooites and Barians realize that the gravitational interaction between their planets will cause them to drop together and collide. It seems that they should get ready for the end of the world. And yet before they riot, get drunk, or tell their spouses that yes, they really *do* look fat in that dress, maybe they should consider the possibility that the two planets will simply hover in place for some amount of time, because that would satisfy conservation of energy. Now the physical implausibility of the hovering solution becomes even more apparent. Not only does one planet have to "decide" at precisely what microsecond to go ahead and fall, but the other planet has to make the same decision at the same instant, or else conservation of energy will be violated. There is no physical process or interaction between the two planets that could perfectly synchronize their "decisions" like this. (The mechanism can't be gravity, because nothing about the gravitational interaction provides any kind of a count-down that would pick out one particular time as the one at which the planets should start moving.)

The key to making sense of all this is to realize that each planet can only "feel" the gravitational field in its own region of space. Its acceleration can only depend on the field, and not on the detailed arrangement of masses elsewhere in the universe that caused that field. Granting this kind of "real" status to fields can be considered as a logically necessary supplement to conservation of energy.

## Automated search for the brachistochrone

See page 95.

```
1  d=.01
2  c1=.61905
3  c2=-.94427
4  a = 1.
5  b = 1.
6  for i in range(100):
7      bestt = 99.
8      for j in range(3):
9          for k in range(3):
10             try_c1 = c1+(j-1)*d
11             try_c2 = c2+(k-1)*d
12             t = timeb(a,b,try_c1,try_c2,100000)
13             if t<bestt :
14                 bestc1 = try_c1
15                 bestc2 = try_c2
16                 bestj = j
17                 bestk = k
```

```

18         bestt = t
19     c1 = bestc1
20     c2 = bestc2
21     c3 = (b-c1*a-c2*a**2)/(a**3)
22     print(c1, c2, c3, bestt)
23     if (bestj == 1) and (bestk == 1) :
24         d = d*.5

```

## Derivation of the steady state for damped, driven oscillations

Using the trig identities for the sine of a sum and cosine of a sum, we can change equation [2] on page 177 into the form

$$\begin{aligned}
 & [(-m\omega^2 + k) \cos \delta - b\omega \sin \delta - F_m/A] \sin \omega t \\
 & + [(-m\omega^2 + k) \sin \delta + b\omega \cos \delta] \cos \omega t = 0.
 \end{aligned}$$

Both the quantities in square brackets must equal zero, which gives us two equations we can use to determine the unknowns  $A$  and  $\delta$ . The results are

$$\begin{aligned}
 \delta &= \tan^{-1} \frac{b\omega}{m\omega^2 - k} \\
 &= \tan^{-1} \frac{\omega\omega_0}{Q(\omega_0^2 - \omega^2)}
 \end{aligned}$$

and

$$\begin{aligned}
 A &= \frac{F_m}{\sqrt{(m\omega^2 - k)^2 + b^2\omega^2}} \\
 &= \frac{F_m}{m\sqrt{(\omega^2 - \omega_0^2)^2 + \omega_0^2\omega^2Q^{-2}}}.
 \end{aligned}$$

## Proofs relating to angular momentum

### Uniqueness of the cross product

The vector cross product as we have defined it has the following properties:

- (1) It does not violate rotational invariance.
- (2) It has the property  $\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$ .
- (3) It has the property  $\mathbf{A} \times (k\mathbf{B}) = k(\mathbf{A} \times \mathbf{B})$ , where  $k$  is a scalar.

**Theorem:** The definition we have given is the only possible method of multiplying two vectors to make a third vector which has these properties, with the exception of trivial redefinitions which just involve multiplying all the results by the same constant or swapping the names of the axes. (Specifically, using a left-hand rule rather than a right-hand rule corresponds to multiplying all the results by  $-1$ .)

**Proof:** We prove only the uniqueness of the definition, without explicitly proving that it has properties (1) through (3).

Using properties (2) and (3), we can break down any vector multiplication  $(A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}) \times (B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}})$  into terms involving cross products of unit vectors.

A “self-term” like  $\hat{\mathbf{x}} \times \hat{\mathbf{x}}$  must either be zero or lie along the  $x$  axis, since any other direction would violate property (1). If it was not zero, then  $(-\hat{\mathbf{x}}) \times (-\hat{\mathbf{x}})$  would have to lie in the opposite

direction to avoid breaking rotational invariance, but property (3) says that  $(-\hat{\mathbf{x}}) \times (-\hat{\mathbf{x}})$  is the same as  $\hat{\mathbf{x}} \times \hat{\mathbf{x}}$ , which is a contradiction. Therefore the self-terms must be zero.

An “other-term” like  $\hat{\mathbf{x}} \times \hat{\mathbf{y}}$  could conceivably have components in the  $x$ - $y$  plane and along the  $z$  axis. If there was a nonzero component in the  $x$ - $y$  plane, symmetry would require that it lie along the diagonal between the  $x$  and  $y$  axes, and similarly the in-the-plane component of  $(-\hat{\mathbf{x}}) \times \hat{\mathbf{y}}$  would have to be along the other diagonal in the  $x$ - $y$  plane. Property (3), however, requires that  $(-\hat{\mathbf{x}}) \times \hat{\mathbf{y}}$  equal  $-(\hat{\mathbf{x}} \times \hat{\mathbf{y}})$ , which would be along the original diagonal. The only way it can lie along both diagonals is if it is zero.

We now know that  $\hat{\mathbf{x}} \times \hat{\mathbf{y}}$  must lie along the  $z$  axis. Since we are not interested in trivial differences in definitions, we can fix  $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ , ignoring peurile possibilities such as  $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = 7\hat{\mathbf{z}}$  or the left-handed definition  $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = -\hat{\mathbf{z}}$ . Given  $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ , the symmetry of space requires that similar relations hold for  $\hat{\mathbf{y}} \times \hat{\mathbf{z}}$  and  $\hat{\mathbf{z}} \times \hat{\mathbf{x}}$ , with at most a difference in sign. A difference in sign could always be eliminated by swapping the names of some of the axes, so ignoring possible trivial differences in definitions we can assume that the cyclically related set of relations  $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ ,  $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ , and  $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$  holds. Since the arbitrary cross-product with which we started can be broken down into these simpler ones, the cross product is uniquely defined.

### The choice of axis theorem

**Theorem:** Suppose a closed system of material particles conserves angular momentum in one frame of reference, with the axis taken to be at the origin. Then conservation of angular momentum is unaffected if the origin is relocated or if we change to a frame of reference that is in constant-velocity motion with respect to the first one. The theorem also holds in the case where the system is not closed, but the total external force is zero.

**Proof:** In the original frame of reference, angular momentum is conserved, so we have  $d\mathbf{L}/dt=0$ . From example 28 on page 284, this derivative can be rewritten as

$$\frac{d\mathbf{L}}{dt} = \sum_i \mathbf{r}_i \times \mathbf{F}_i,$$

where  $\mathbf{F}_i$  is the total force acting on particle  $i$ . In other words, we’re adding up all the torques on all the particles.

By changing to the new frame of reference, we have changed the position vector of each particle according to  $\mathbf{r}_i \rightarrow \mathbf{r}_i + \mathbf{k} - \mathbf{u}t$ , where  $\mathbf{k}$  is a constant vector that indicates the relative position of the new origin at  $t = 0$ , and  $\mathbf{u}$  is the velocity of the new frame with respect to the old one. The forces are all the same in the new frame of reference, however. In the new frame, the rate of change of the angular momentum is

$$\begin{aligned} \frac{d\mathbf{L}}{dt} &= \sum_i (\mathbf{r}_i + \mathbf{k} - \mathbf{u}t) \times \mathbf{F}_i \\ &= \sum_i \mathbf{r}_i \times \mathbf{F}_i + (\mathbf{k} - \mathbf{u}t) \times \sum_i \mathbf{F}_i. \end{aligned}$$

The first term is the expression for the rate of change of the angular momentum in the original frame of reference, which is zero by assumption. The second term vanishes by Newton’s third law; since the system is closed, every force  $\mathbf{F}_i$  cancels with some force  $\mathbf{F}_j$ . (If external forces act, but they add up to zero, then the sum can be broken up into a sum of internal forces and a sum of external forces, each of which is zero.) The rate of change of the angular momentum is therefore zero in the new frame of reference.

## The spin theorem

**Theorem:** An object's angular momentum with respect to some outside axis A can be found by adding up two parts:

- (1) The first part is the object's angular momentum found by using its own center of mass as the axis, i.e. the angular momentum the object has because it is spinning.
- (2) The other part equals the angular momentum that the object would have with respect to the axis A if it had all its mass concentrated at and moving with its center of mass.

**Proof:** Let  $\mathbf{r}_{cm}$  be the position of the center of mass. The total angular momentum is

$$\mathbf{L} = \sum_i \mathbf{r}_i \times \mathbf{p}_i,$$

which can be rewritten as

$$\mathbf{L} = \sum_i (\mathbf{r}_{cm} + \mathbf{r}_i - \mathbf{r}_{cm}) \times \mathbf{p}_i,$$

where  $\mathbf{r}_i - \mathbf{r}_{cm}$  is particle  $i$ 's position relative to the center of mass. We then have

$$\begin{aligned} \mathbf{L} &= \mathbf{r}_{cm} \times \sum_i \mathbf{p}_i + \sum_i (\mathbf{r}_i - \mathbf{r}_{cm}) \times \mathbf{p}_i \\ &= \mathbf{r}_{cm} \times \mathbf{p}_{total} + \sum_i (\mathbf{r}_i - \mathbf{r}_{cm}) \times \mathbf{p}_i \\ &= \mathbf{r}_{cm} \times m_{total} \mathbf{v}_{cm} + \sum_i (\mathbf{r}_i - \mathbf{r}_{cm}) \times \mathbf{p}_i. \end{aligned}$$

The first and second terms in this expression correspond to the quantities (2) and (1), respectively.

## Different Forms of Maxwell's Equations

First we reproduce Maxwell's equations as stated on page 701, in integral form, using the SI (meter-kilogram-second) system of units, with the coupling constants written in terms of  $k$  and  $c$ :

$$\begin{aligned} \Phi_E &= 4\pi k q_{in} \\ \Phi_B &= 0 \\ \Gamma_E &= -\frac{\partial \Phi_B}{\partial t} \\ c^2 \Gamma_B &= \frac{\partial \Phi_E}{\partial t} + 4\pi k I_{through} \end{aligned}$$

Homework problem 39 on page 732 deals with rewriting these in terms of  $\epsilon_o = 1/4\pi k$  and  $\mu_o = 4\pi k/c^2$  rather than  $k$  and  $c$ .

For the reader who has been studying the optional sections giving Maxwell's equations in differential form, here is a summary:

$$\begin{aligned}
\operatorname{div} \mathbf{E} &= 4\pi k\rho \\
\operatorname{div} \mathbf{B} &= 0 \\
\operatorname{curl} \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
c^2 \operatorname{curl} \mathbf{B} &= \frac{\partial \mathbf{E}}{\partial t} + 4\pi k \mathbf{j}
\end{aligned}$$

Although all engineering and most scientific work these days is done in the SI (mks) system, one may still encounter the older cgs (centimeter-gram-second) system, especially in astronomy and particle physics. The mechanical units in this system include the dyne ( $\text{g}\cdot\text{cm}/\text{s}^2$ ) for force, and the erg ( $\text{g}\cdot\text{cm}^2/\text{s}^2$ ) for energy. The system is extended to electrical units by taking  $k = 1$  as a matter of definition, so the Coulomb force law is  $F = q_1q_2/r^2$ . This equation indirectly defines a unit of charge called the electrostatic unit, with  $1 \text{ C} = 2.998 \times 10^9 \text{ esu}$ , the factor of 2.998 arising from the speed of light. The unit of voltage is the statvolt,  $1 \text{ statvolt} = 299.8 \text{ V}$ . In this system, the electric and magnetic fields have the same units, dynes/esu, but to avoid confusion, magnetic fields are normally written using the equivalent unit of gauss,  $1 \text{ gauss} = 1 \text{ dyne/esu} = 10^{-4} \text{ T}$ . The force on a charged particle is  $\mathbf{F} = q\mathbf{E} + q\frac{\mathbf{v}}{c} \times \mathbf{B}$ , which differs from the mks version by the  $1/c$  factor in the magnetic term. Maxwell's equations are:

$$\begin{aligned}
\Phi_E &= 4\pi q_{in} \\
\Phi_B &= 0 \\
\Gamma_E &= -\frac{1}{c} \frac{\partial \Phi_B}{\partial t} \\
\Gamma_B &= \frac{1}{c} \frac{\partial \Phi_E}{\partial t} + \frac{4\pi}{c} I_{through}
\end{aligned}$$

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# Appendix 4: Hints and Solutions

## Hints

### Hints for Chapter 2

**Page 120, problem 16:** You can use either the chain-rule technique from page 83 or the technique prescribed in problem 15 on p. 120. The positions and velocities of the two masses are related to each other, and you'll need to use this relationship to eliminate one mass's position and velocity and get everything in terms of the other mass's position and velocity. The relationship between the two positions will involve some extraneous variables like the length of the string, which won't have any effect on your final result.

**Page 120, problem 17:** This is similar to problem 16, but you're trying to find the combination of masses that will result in *zero* acceleration. In this problem, the distance dropped by one weight is different from, but still related to, the distance by which the other weight rises. Try relating the heights of the two weights to each other, so you can get the total gravitational energy in terms of only one of these heights.

**Page 120, problem 18:** This is similar to problem 17, in that you're looking for a setup that will give zero acceleration, and the distance the middle weight rises or falls is not the same as the distance the other two weights fall or rise. The simplest approach is to get the three heights in terms of  $\theta$ , so that you can write the gravitational energy in terms of  $\theta$ .

**Page 120, problem 19:** This is very similar to problems 16 and 17.

**Page 120, problem 20:** The first two parts can be done more easily by setting  $a = 1$ , since the value of  $a$  only changes the distance scale.

**Page 121, problem 22:** The condition for a circular orbit contains three unknowns,  $v$ ,  $g$ , and  $r$ , so you can't just solve it for  $r$ . You'll need more equations to make three equations in three unknowns. You'll need a relationship between  $g$  and  $r$ , and also a relationship between  $v$  and  $r$  that uses the given fact that it's supposed to take 24 hours for an orbit.

**Page 121, problem 25:** What does the total energy have to be if the projectile's velocity is exactly escape velocity? Write down conservation of energy, change  $v$  to  $dr/dt$ , separate the variables, and integrate.

**Page 121, problem 26:** The analytic approach is a little cumbersome, although it can be done by using approximations like  $1/\sqrt{1+\epsilon} \approx 1 - (1/2)\epsilon$ . A more straightforward, brute-force method is simply to write a computer program that calculates  $U/m$  for a given point in spherical coordinates. By trial and error, you can fairly rapidly find the  $r$  that gives a desired value of  $U/m$ .

**Page 123, problem 33:** Use calculus to find the minimum of  $U$ .

**Page 123, problem 35:** The spring constant of this spring,  $k$ , is *not* the quantity you need in the equation for the period. What you need in that equation is the second derivative of the spring's energy with respect to the position of the thing that's oscillating. You need to start by finding the energy stored in the spring as a function of the vertical position,  $y$ , of the mass.



This is similar to example 23 on page 116.

**Page 124, problem 37:** The variables  $x_1$  and  $x_2$  will adjust themselves to reach an equilibrium. Write down the total energy in terms of  $x_1$  and  $x_2$ , then eliminate one variable, and find the equilibrium value of the other. Finally, eliminate both  $x_1$  and  $x_2$  from the total energy, getting it just in terms of  $b$ .

### Hints for Chapter 3

**Page 221, problem 20:** Write down two equations, one for Newton's second law applied to each object. Solve these for the two unknowns  $T$  and  $a$ .

**Page 225, problem 41:** The whole expression for the amplitude has maxima where the stuff inside the square root is at a minimum, and vice versa, so you can save yourself a lot of work by just working on the stuff inside the square root. For normal, large values of  $Q$ , there are two extrema, one at  $\omega = 0$  and one at resonance; one of these is a maximum and one is a minimum. You want to find out at what value of  $Q$  the zero-frequency extremum switches over from being a maximum to being a minimum.

**Page 230, problem 69:** You can use the geometric interpretation of the dot product.

**Page 230, problem 70:** The easiest way to do this problem is to use two different coordinate systems: one that's tilted to coincide with the upper slope, and one that's tilted to coincide with the lower one.

### Hints for Chapter 4

**Page 288, problem 8:** The choice of axis theorem only applies to a closed system, or to a system acted on by a total force of zero. Even if the box is not going to rotate, its center of mass is going to accelerate, and this can still cause a change in its angular momentum, unless the right axis is chosen. For example, if the axis is chosen at the bottom right corner, then the box will start accumulating clockwise angular momentum, even if it is just accelerating to the right without rotating. Only by choosing the axis at the center of mass (or at some other point on the same horizontal line) do we get a constant, zero angular momentum.

**Page 289, problem 11:** There are four forces on the wheel at first, but only three when it lifts off. Normal forces are always perpendicular to the surface of contact. Note that the corner of the step cannot be perfectly sharp, so the surface of contact for this force really coincides with the surface of the wheel.

**Page 289, problem 12:** How is this different from the case where you whirl a rock in a circle on a string and gradually pull in the string?

**Page 292, problem 24:** The acceleration and the tension in the string are unknown. Use  $\tau = dL/dt$  and  $F = ma$  to determine these two unknowns.

**Page 294, problem 35:** You'll need the result of problem 19 in order to relate the energy and angular momentum of a rigidly rotating body. Since this relationship involves a variable raised to a power, you can't just graph the data and get the moment of inertia directly. One way to get around this is to manipulate one of the variables to make the graph linear. Here is an example of this technique from another context. Suppose you were given a table of the masses,  $m$ , of cubical pieces of wood, whose sides had various lengths,  $b$ . You want to find a best-fit value for the density of the wood. The relationship is  $m = \rho b^3$ . The graph of  $m$  versus  $b$  would be a curve, and you would not have any easy way to get the density from such a graph. But by graphing  $m$  versus  $b^3$ , you can produce a graph that is linear, and whose slope equals the density.

## Hints for Chapter 6

**Page 381, problem 4:** How could you change the values of  $x$  and  $t$  so that the value of  $y$  would remain the same? What would this represent physically?

**Page 383, problem 12:** The answers to the two parts are not the same.

**Page 382, problem 8:** (a) The most straightforward approach is to apply the equation  $\partial^2 y / \partial t^2 = (T/\mu)\partial^2 y / \partial x^2$ . Although this equation was developed in the main text in the context of a straight string with a curvy wave on it, it works just as well for a circular loop; the left-hand side is simply the inward acceleration of any point on the rope. Note, however, that we've been assuming the string was (at least approximately) parallel to the  $x$  axis, which will only be true if you choose a specific value of  $x$ . You need to get an equation for  $y$  in terms of  $x$  in order to evaluate the right-hand side.

## Hints for Chapter 7

**Page 449, problem 28:** Apply the equivalence principle.

## Hints for Chapter 10

**Page 640, problem 15:** Use the approximation  $(1 + \epsilon)^p \approx 1 + p\epsilon$ , which is valid for small  $\epsilon$ .

**Page 644, problem 31:** The math is messy if you put the origin of your polar coordinates at the center of the disk. It comes out much simpler if you put the origin at the edge, right on top of the point at which we're trying to compute the voltage.

**Page 643, problem 26:** Since we have  $t \ll r$ , the volume of the membrane is essentially the same as if it was unrolled and flattened out, and the field's magnitude is nearly constant.

**Page 643, problem 25:** First find the energy stored in a spherical shell extending from  $r$  to  $r + dr$ , then integrate to find the total energy.

**Page 648, problem 59:** We have  $D = ql$  and  $F_x = qbl = D\partial E_x / \partial x$ , which, as claimed, is consistent with the result of example 6 on p. 573 and depends on  $q$  and  $\ell$  only via  $D$ .

## Hints for Chapter 11

**Page 729, problem 24:** A stable system has low energy; energy would have to be added to change its configuration.

**Page 733, problem 41:** We're ignoring the fact that the light consists of little wavepackets, and imagining it as a simple sine wave. But wait, there's more good news! The energy density depends on the squares of the fields, which means the squares of some sine waves. Well, when you square a sine wave that varies from  $-1$  to  $+1$ , you get a sine wave that goes from  $0$  to  $+1$ , and the average value of that sine wave is  $1/2$ . That means you don't have to do an integral like  $U = \int (dU/dV) dV$ . All you have to do is throw in the appropriate factor of  $1/2$ , and you can pretend that the fields have their constant values  $\tilde{\mathbf{E}}$  and  $\tilde{\mathbf{B}}$  everywhere.

**Page 733, problem 42:** Use Faraday's law, and choose an Ampèrian surface that is a disk of radius  $R$  sandwiched between the plates.

**Page 735, problem 51:** (a) Magnetic fields are created by currents, so once you've decided how currents behave under time-reversal, you can figure out how magnetic fields behave.

## Hints for Chapter 12

**Page 818, problem 60:** Expand  $\sin \theta$  in a Taylor series around  $\theta = 90^\circ$ .

## Answers to Self-Checks

### Answers to Self-Checks for Chapter 0

**Page 15:** If only he has the special powers, then his results can never be reproduced.

**Page 17:** They would have had to weigh the rays, or check for a loss of weight in the object from which they were have emitted. (For technical reasons, this was not a measurement they could actually do, hence the opportunity for disagreement.)

**Page 25:** A dictionary might define “strong” as “possessing powerful muscles,” but that’s not an operational definition, because it doesn’t say how to measure strength numerically. One possible operational definition would be the number of pounds a person can bench press.

**Page 28:** A microsecond is 1000 times longer than a nanosecond, so it would seem like 1000 seconds, or about 20 minutes.

**Page 29:** Exponents have to do with multiplication, not addition. The first line should be 100 times longer than the second, not just twice as long.

**Page 32:** The various estimates differ by 5 to 10 million. The CIA’s estimate includes a ridiculous number of gratuitous significant figures. Does the CIA understand that every day, people in are born in, die in, immigrate to, and emigrate from Nigeria?

**Page 33:** (1) 4; (2) 2; (3) 2

**Page 36:**  $1 \text{ yd}^2 \times (3 \text{ ft}/1 \text{ yd})^2 = 9 \text{ ft}^2$   
 $1 \text{ yd}^3 \times (3 \text{ ft}/1 \text{ yd})^3 = 27 \text{ ft}^3$

**Page 42:**  $C_1/C_2 = (w_1/w_2)^4$

### Answers to Self-Checks for Chapter 1

**Page 58:** The stream has to spread out. When the velocity becomes zero, it seems like the cross-sectional area has to become infinite. In reality, this is the point where the water turns around and comes back down. The infinity isn’t real; it occurs mathematically because we used a simplified model of the the stream of water, assuming, for instance, that the water’s velocity is always straight up.

**Page 60:** A positive  $\Delta x$  means the object is moving in the same direction as the positive  $x$  axis. A negative  $\Delta x$  means it’s going the opposite direction.

**Page 66:** (1) The effect only occurs during blastoff, when their velocity is changing. Once the rocket engines stop firing, their velocity stops changing, and they no longer feel any effect. (2) It is only an observable effect of your motion relative to the air.

**Page 68:** Galilean relativity says that experiments can’t come out differently just because they’re performed while in motion. The tilting of the surface tells us the train is accelerating, but it doesn’t tell us anything about the train’s velocity at that instant. The person in the train might say the bottle’s velocity was zero (but changing), whereas a person working in a reference frame attached to the dirt outside says it’s moving; they don’t agree on velocities. They *do* agree on accelerations. The person in the train has to agree that the train is accelerating, since otherwise there’s no reason for the funny tilting effect.

**Page 69:** Yes. In U.S. currency, for instance, the quantum of money is one cent.

### Answers to Self-Checks for Chapter 2

**Page 90:** There are two reasonable possibilities we could imagine — neither of which ends up making much sense — if we insist on the straight-line trajectory. (1) If the car has constant speed along the line, then in the \* frame we see it going straight down at constant speed. It

makes sense that it goes straight down in the \* frame of reference, since in that frame it was never moving horizontally, and there's no reason for it to start. However, it doesn't make sense that it goes down with constant speed, since falling objects are supposed to speed up the whole time they fall. This violates both Galilean relativity and conservation of energy. (2) If it's speeding up and moving along a diagonal line in the original frame, then it might be conserving energy in one frame or the other. But if it's speeding up along a line, then as seen in the original frame, both its vertical motion and its horizontal motion must be speeding up. If its horizontal velocity is increasing in the original frame, then it can't be zero and remain zero in the \* frame. This violates Galilean relativity, since in the \* frame the car apparently starts moving sideways for no reason.

### Answers to Self-Checks for Chapter 3

**Page 143:** By shifting his weight around, he can cause the center of mass not to coincide with the geometric center of the wheel.

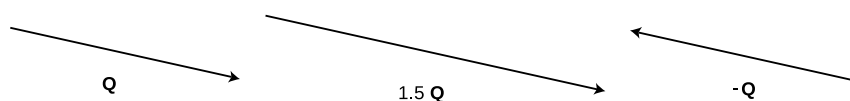
**Page 151:** (1) This is motion, not force. (2) This is a description of how the sub is able to get the water to produce a forward force on it. (3) The sub runs out of energy, not force.

**Page 153:** (1) It's kinetic friction, because her uniform is sliding over the dirt. (2) It's static friction, because even though the two surfaces are moving relative to the landscape, they're not slipping over each other. (3) Only kinetic friction creates heat, as when you rub your hands together. If you move your hands up and down together without sliding them across each other, no heat is produced by the static friction.

**Page 152:** Frictionless (or nearly frictionless) ice can certainly make a normal force, since otherwise a hockey puck would sink into the ice. Friction is not possible without a normal force, however: we can see this from the equation, or from common sense, e.g. while sliding down a rope you don't get any friction unless you grip the rope.

**Page 154:** (1) Normal forces are always perpendicular to the surface of contact, which means right or left in this figure. Normal forces are repulsive, so the cliff's force on the feet is to the right, i.e., away from the cliff. (2) Frictional forces are always parallel to the surface of contact, which means right or left in this figure. Static frictional forces are in the direction that would tend to keep the surfaces from slipping over each other. If the wheel was going to slip, its surface would be moving to the left, so the static frictional force on the wheel must be in the direction that would prevent this, i.e., to the right. This makes sense, because it is the static frictional force that accelerates the dragster. (3) Normal forces are always perpendicular to the surface of contact. In this diagram, that means either up and to the left or down and to the right. Normal forces are repulsive, so the ball is pushing the bat away from itself. Therefore the ball's force is down and to the right on this diagram.

**Page 195:**



**Page 171:** The dashed lines on the graph are about twice as far apart in the second cycle compared to the first, so the amplitude has doubled. For sufficiently small oscillations around an equilibrium with  $x = 0$  and  $U(0) = 0$ , it's always a good approximation to take  $U \propto x^2$ , so the energy is proportional to the square of the amplitude; this is a general fact about all oscillations, provided that the amplitude is small. Since the amplitude doubled, the energy quadrupled.

**Page 174:** The two graphs start off with the same amplitude, but the solid curve loses amplitude more rapidly. For a given time,  $t$ , the quantity  $e^{-ct}$  is apparently smaller for the solid curve, meaning that  $ct$  is greater. The solid curve has the higher value of  $c$ .

**Page 176:** A decaying exponential never dies out to zero in any finite amount of time.

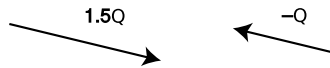
**Page 180:** In the expression

$$A = \frac{F_m}{m\sqrt{(\omega^2 - \omega_0^2)^2 + \omega_0^2\omega^2Q^{-2}}}$$

from page 952, substituting  $\omega = \omega_0$  makes the first term inside the square root vanish, which should make the denominator pretty small, thereby producing a pretty big amplitude. In the limit of  $Q = \infty$ ,  $Q^{-2} = 0$ , so the second term vanishes, and  $\omega = \omega_0$  actually produces an infinite amplitude. For values of  $Q$  that are large but finite, we still expect to get resonance pretty close to  $\omega = \omega_0$ . Setting  $\omega = \omega_0$  in the finite- $Q$  case, the first term vanishes, we can simplify the square root, and the result ends up being  $A \propto 1/\sqrt{Q^{-2}} \propto Q$ . This is only an approximation, because we had to assume early on that  $Q$  was large.

**Page 194:**  $\mathbf{F} = ma$

**Page ??:**



#### Answers to Self-Checks for Chapter 4

**Page 256:** Torques 1, 2, and 4 all have the same sign, because they are trying to twist the wrench clockwise. The sign of 3 is opposite to the signs of 1, 2, and 4. The magnitude of 3 is the greatest, since it has a large  $r$  and the force is nearly all perpendicular to the wrench. Torques 1 and 2 are the same because they have the same values of  $r$  and  $F_{\perp}$ . Torque 4 is the smallest, due to its small  $r$ .

**Page 265:** One person's  $\theta$ - $t$  graph would simply be shifted up or down relative to the others. The derivative equals the slope of the tangent line, and this slope isn't changed when you shift the graph, so both people would agree on the angular velocity.

**Page 267:** Reversing the direction of  $\omega$  also reverses the direction of motion, and this is reflected by the relationship between the plus and minus signs. In the equation for the radial acceleration,  $\omega$  is squared, so even if  $\omega$  is negative, the result is positive. This makes sense because the acceleration is always inward in circular motion, never outward.

**Page 279:** All the rotations around the  $x$  axis give  $\omega$  vectors along the positive  $x$  axis (thumb pointing along positive  $x$ ), and all the rotations about the  $y$  axis have  $\omega$  vectors with positive  $y$  components.

**Page 282:** For example, if we take  $(\mathbf{A} \times \mathbf{B})_x = A_yB_z - B_yA_z$  and reverse the A's and B's, we get  $(\mathbf{B} \times \mathbf{A})_x = B_yA_z - A_yB_z$ , which is just the negative of the original expression.

#### Answers to Self-Checks for Chapter 5

**Page 303:** Solids can exert shear forces. A solid could be in an equilibrium in which the shear forces were canceling the forces due to sideways pressure gradients. For example, if I push on a brick wall, it will give by perhaps a millionth of an inch, but it will reach an equilibrium, in which the shear forces cancel out the effect of the pressure gradient.

**Page 303:** (1) Not valid. The equation only applies to fluids. (2) Valid. The density of the air is nearly constant between the top and bottom of the building. (3) Not valid. There is a large difference in the density of the air between the top and the bottom of the mountain. (4) Not valid, because  $g$  isn't constant throughout the interior of the earth. (5) Not valid, because the air is flowing around the wing. The air is accelerating, so it is not in equilibrium.

**Page 324:** Heating the gas at constant pressure requires adding heat to it, which increases its entropy. To increase the gas's pressure while keeping its temperature constant, we would have to compress it, which would give it a smaller volume to inhabit, and therefore fewer possible positions for each atom. The whole thing has to be proportional to  $n$  because entropy is additive.

### Answers to Self-Checks for Chapter 6

**Page 346:** The leading edge is moving up, the trailing edge is moving down, and the top of the hump is motionless for one instant.

**Page 365:** The energy of a wave is usually proportional to the square of the amplitude. Squaring a negative number gives a positive result, so the energy is the same.

**Page 366:** A substance is invisible to sonar if the speed of sound waves in it is the same as in water. Reflections occur only at boundaries between media in which the wave speed is different.

**Page 367:** No. A material object that loses kinetic energy slows down, but a wave is not a material object. The velocity of a wave ordinarily only depends on the medium, not on the amplitude. The speed of soft sound, for example, is the same as the speed of loud sound.

**Page 374:** No. To get the best possible interference, the thickness of the coating must be such that the second reflected wave train lags behind the first by an integer number of wavelengths. Optimal performance can therefore only be produced for one specific color of light. The typical greenish color of the coatings shows that it does the worst job for green light.

**Page 375:** The period is the time required to travel a distance  $2L$  at speed  $v$ ,  $T = 2L/v$ . The frequency is  $f = 1/T = v/2L$ .

**Page 379:** The wave pattern will look like this:  $\otimes$  Three quarters of a wavelength fit in the tube, so the wavelength is three times shorter than that of the lowest-frequency mode, in which one quarter of a wave fits. Since the wavelength is smaller by a factor of three, the frequency is three times higher. Instead of  $f_0, 2f_0, 3f_0, 4f_0, \dots$ , the pattern of wave frequencies of this air column goes  $f_0, 3f_0, 5f_0, 7f_0, \dots$

### Answers to Self-Checks for Chapter 7

**Page 394:** At  $v = 0$ , we get  $\gamma = 1$ , so  $t = T$ . There is no time distortion unless the two frames of reference are in relative motion.

**Page 416:** The total momentum is zero before the collision. After the collision, the two momenta have reversed their directions, but they still cancel. Neither object has changed its kinetic energy, so the total energy before and after the collision is also the same.

**Page 414:** Both the time axis and the position axis have been turned around. Flipping the time axis means that the roles of transmitter and receiver have been swapped, and it also means that Alice and Betty are approaching one another rather than receding. The time experienced by the receiving observer is now the longer one, so the Doppler-shift factor has been inverted: the receiver now measures a Doppler shift of  $1/2$  rather than  $2$  in frequency.

**Page 423:** At  $v = 0$ , we have  $\gamma = 1$ , so the mass-energy is  $mc^2$  as claimed. As  $v$  approaches  $c$ ,  $\gamma$  approaches infinity, so the mass energy becomes infinite as well.

## Answers to Self-Checks for Chapter 8

**Page 463:** Either type can be involved in either an attraction or a repulsion. A positive charge could be involved in either an attraction (with a negative charge) or a repulsion (with another positive), and a negative could participate in either an attraction (with a positive) or a repulsion (with a negative).

**Page 464:** It wouldn't make any difference. The roles of the positive and negative charges in the paper would be reversed, but there would still be a net attraction.

**Page 474:** Yes. In U.S. currency, the quantum of money is the penny.

**Page 494:** Thomson was accelerating electrons, which are negatively charged. This apparatus is supposed to accelerated atoms with one electron stripped off, which have positive net charge. In both cases, a particle that is between the plates should be attracted by the forward plate and repelled by the plate behind it.

**Page 502:** The hydrogen-1 nucleus is simple a proton. The binding energy is the energy required to tear a nucleus apart, but for a nucleus this simple there is nothing to tear apart.

## Answers to Self-Checks for Chapter 9

**Page 531:** The large amount of power means a high rate of conversion of the battery's chemical energy into heat. The battery will quickly use up all its energy, i.e. "burn out."

## Answers to Self-Checks for Chapter 10

**Page 569:** The reasoning is exactly analogous to that used in example 1 on page 567 to derive an equation for the gravitational field of the earth. The field is  $F/q_t = (kQq_t/r^2)/q_t = kQ/r^2$ .

**Page 576:**

$$\begin{aligned} E_x &= -\frac{dV}{dx} \\ &= -\frac{d}{dx} \left( \frac{kQ}{r} \right) \\ &= \frac{kQ}{r^2} \end{aligned}$$

**Page 577:** (a) The voltage (height) increases as you move to the east or north. If we let the positive  $x$  direction be east, and choose positive  $y$  to be north, then  $dV/dx$  and  $dV/dy$  are both positive. This means that  $E_x$  and  $E_y$  are both negative, which makes sense, since the water is flowing in the negative  $x$  and  $y$  directions (south and west).

(b) The electric fields are all pointing away from the higher ground. If this was an electrical map, there would have to be a large concentration of charge all along the top of the ridge, and especially at the mountain peak near the south end.

**Page 589:** (a) The energy density depends on  $\mathbf{E} \cdot \mathbf{E}$ , which equals  $E_x^2 + E_y^2 + E_z^2$ .

(b) Since  $E_x$  is squared, reversing its sign has no effect on the energy density. This makes sense, because otherwise we'd be saying that the positive and negative  $x$  axes in space were somehow physically different in their behavior, which would violate the symmetry of space.

Page 589:

$$\begin{aligned} \text{N}^{-1}\text{m}^{-2}\text{C}^2\text{V}^2\text{m}^{-2}\text{m}^2\text{m} &= \text{N}^{-1}\text{m}^{-1}\text{C}^2\text{V}^2 \\ &= \text{N}^{-1}\text{m}^{-1}\text{J}^2 \\ &= \text{J}^{-1}\text{J}^2 \\ &= \text{J} \end{aligned}$$

**Page 598:** Yes. The mass has the same kinetic energy regardless of which direction it's moving. Friction converts mechanical energy into heat at the same rate whether the mass is sliding to the right or to the left. The spring has an equilibrium length, and energy can be stored in it either by compressing it ( $x < 0$ ) or stretching it ( $x > 0$ ).

**Page 599:** Velocity,  $v$ , is the derivative of position,  $x$ , with respect to time. This is exactly analogous to  $I = dq/dt$ .

**Page 614:** The impedance depends on the frequency at which the capacitor is being driven. It isn't just a single value for a particular capacitor.

**Page 609:** Say we're looking for  $u = \sqrt{z}$ , i.e., we want a number  $u$  that, multiplied by itself, equals  $z$ . Multiplication multiplies the magnitudes, so the magnitude of  $u$  can be found by taking the square root of the magnitude of  $z$ . Since multiplication also adds the arguments of the numbers, squaring a number doubles its argument. Therefore we can simply divide the argument of  $z$  by two to find the argument of  $u$ . This results in one of the square roots of  $z$ . There is another one, which is  $-u$ , since  $(-u)^2$  is the same as  $u^2$ . This may seem a little odd: if  $u$  was chosen so that doubling its argument gave the argument of  $z$ , then how can the same be true for  $-u$ ? Well for example, suppose the argument of  $z$  is  $4^\circ$ . Then  $\arg u = 2^\circ$ , and  $\arg(-u) = 182^\circ$ . Doubling 182 gives 364, which is actually a synonym for 4 degrees.

**Page 613:** Only  $\cos(6t - 4)$  can be represented by a complex number. Although the graph of  $\cos^2 t$  does have a sinusoidal shape, it varies between 0 and 1, rather than  $-1$  and 1, and there is no way to represent that using complex numbers. The function  $\tan t$  doesn't even have a sinusoidal shape.

**Page 626:** The quantity  $4\pi kq_{in}$  is now negative, so we'd better get a negative flux on the other side of Gauss' theorem. We do, because each field vector  $\mathbf{E}_j$  is inward, while the corresponding area vector,  $\mathbf{A}_j$ , is outward. Vectors in opposite directions make negative dot products.

### Answers to Self-Checks for Chapter 11

**Page 664:** For instance, imagine a small sphere around the negative charge, which we would sketch on the two-dimensional paper as a circle. The field points inward at every point on the sphere, so all the contributions to the flux are negative. There is no cancellation, and the total flux is negative, which is consistent with Gauss' law, since the sphere encloses a negative charge. Copying the same surface onto the field of the bar magnet, however, we find that there is inward flux on the top and outward flux on the bottom, where the surface is inside the magnet. According to Gauss' law for magnetism, these cancel exactly, which is plausible based on the figure.

**Page 662:** From the top panel of the figure, where the magnetic field is turned off, we can see that the beam leaves the cathode traveling upward, so in the bottom figure the electrons must be circling in the counterclockwise direction. To produce circular motion, the force must be



towards the center of the circle. We can arbitrarily pick a point on the circle at which to analyze the vectors — let's pick the right-hand side. At this point, the velocity vector of the electrons is upward. Since the electrons are negatively charged, the force  $q\mathbf{v} \times \mathbf{B}$  is given by  $-\mathbf{v} \times \mathbf{B}$ , not  $+\mathbf{v} \times \mathbf{B}$ . Circular orbits are produced when the motion is in the plane perpendicular to the field, so the field must be either into or out of the page. If the field was into the page, the right-hand rule would give  $\mathbf{v} \times \mathbf{B}$  to the left, which is towards the center, but the force would be in the direction of  $-\mathbf{v} \times \mathbf{B}$ , which would be outwards. The field must be out of the page.

**Page 668:** Plugging  $z = 0$  into the equation gives  $B_z = 4kI/c^2h$ . This is simply twice the field of a single wire at a distance  $h$ . At this location, the fields contributed by the two wires are parallel, so vector addition simply gives a vector twice as strong.

**Page 680:** The circulation around the Ampèrian surface we used was counterclockwise, since the field on the bottom was to the right. Applying the right-hand rule, the current  $I_{\text{through}}$  must have been out of the page at the top of the solenoid, and into the page at the bottom.

**Page 680:** The quantity  $\ell$  came in because we set  $\eta = NI/\ell$ . Based on that, it's clear that  $\ell$  represents the length of the solenoid, not the length of the wire.

**Page 681:** Doubling the radius of the solenoid would mean that every distance in the problem would be doubled, which would tend to make the fields weaker, since fields fall off with distance. However, doubling the radius would also mean that we had twice as much wire, and therefore twice as many moving charges to create magnetic fields. Since the magnetic field of a wire falls off like  $1/r$ , it's not surprising that the first effect amounts to exactly a factor of  $1/2$ , which is exactly enough to cancel out the factor of 2 from the second effect.

**Page 693:** Unless the engine is already turning over, the permanent magnet isn't spinning, so there is no change in the magnetic field. Only a changing magnetic field creates an induced electric field.

**Page 698:** Let's get all the electrical units in terms of Teslas. Electric field units can be expressed as  $\text{T} \cdot \text{m}/\text{s}$ . The circulation of the electric field has units of electric field multiplied by distance, or  $\text{T} \cdot \text{m}^2/\text{s}$ . On the right side, the derivative  $\partial\mathbf{B}/\partial t$  has units of  $\text{T}/\text{s}$ , and multiplying this by area gives units of  $\text{T} \cdot \text{m}^2/\text{s}$ , just like on the left side.

**Page 716:** An (idealized) battery is a circuit element that always maintains the same voltage difference across itself, so by the loop rule, the voltage difference across the capacitor must remain unchanged, even while the dielectric is being withdrawn. The bound charges on the surfaces of the dielectric have been attracting the free charges in the plates, causing them to charge up more than they ordinarily would have. As the dielectric is withdrawn, the capacitor will be partially discharged, and we will observe a current in the ammeter. Since the dielectric is attracted to the plates, positive work is done in extracting it, indicating that there must be an increase in the electrical energy stored in the capacitor. This may seem paradoxical, since the energy stored in a capacitor is  $(1/2)CV^2$ , and we are decreasing the capacitance. However, the energy  $(1/2)CV^2$  is calculated in terms of the work required to deposit the free charge on the plates. In addition to this energy, there is also energy stored in the dielectric itself. By moving its bound charges farther away from the free charges in the plates, to which they are attracted, we have increased their electrical energy. This energy of the bound charges is inaccessible to the electric circuit.

## Answers to Self-Checks for Chapter 12

**Page 752:** Only 1 is correct. If you draw the normal that bisects the solid ray, it also bisects

the dashed ray.

**Page 756:** You should have found from your ray diagram that an image is still formed, and it has simply moved down the same distance as the real face. However, this new image would only be visible from high up, and the person can no longer see his own image.

**Page 761:** Increasing the distance from the face to the mirror has decreased the distance from the image to the mirror. This is the opposite of what happened with the virtual image.

**Page 771:** At the top of the graph,  $d_i$  approaches infinity when  $d_o$  approaches  $f$ . Interpretation: the rays just barely converge to the right of the mirror.

On the far right,  $d_i$  approaches  $f$  as  $d_o$  approaches infinity; this is the definition of the focal length.

At the bottom,  $d_i$  approaches negative infinity when  $d_o$  approaches  $f$  from the other side. Interpretation: the rays don't quite converge on the right side of the mirror, so they appear to have come from a virtual image point very far to the left of the mirror.

**Page 780:** (1) If  $n_1$  and  $n_2$  are equal, Snell's law becomes  $\sin \theta_1 = \sin \theta_2$ , which implies  $\theta_1 = \theta_2$ , since both angles are between 0 and  $90^\circ$ . The graph would be a straight line along the diagonal of the graph. (2) The graph is farthest from the diagonal when the angles are large, i.e., when the ray strikes the interface at a grazing angle.

**Page 785:** (1) In 1, the rays cross the image, so it's real. In 2, the rays only appear to have come from the image point, so the image is virtual. (2) A ray is always closer to the normal in the medium with the higher index of refraction. The first left turn makes the ray closer to the normal, which is what should happen in glass. The second left turn makes the ray farther from the normal, and that's what should happen in air. (3) Take the topmost ray as an example. It will still take two right turns, but since it's entering the lens at a steeper angle, it will also leave at a steeper angle. Tracing backward to the image, the steeper lines will meet closer to the lens.

**Page 793:** It would have to have a wavelength on the order of centimeters or meters, the same distance scale as that of your body. These would be microwaves or radio waves. (This effect can easily be noticed when a person affects a TV's reception by standing near the antenna.) None of this contradicts the correspondence principle, which only states that the wave model must agree with the ray model when the ray model is applicable. The ray model is not applicable here because  $\lambda/d$  is on the order of 1.

**Page 795:** At this point, both waves would have traveled nine and a half wavelengths. They would both be at a negative extreme, so there would be constructive interference.

**Page 799:** Judging by the distance from one bright wave crest to the next, the wavelength appears to be about  $2/3$  or  $3/4$  as great as the width of the slit.

**Page 800:** Since the wavelengths of radio waves are thousands of times longer, diffraction causes the resolution of a radio telescope to be thousands of times worse, all other things being equal. (To compensate for the wavelength, it's desirable to make the telescope very large, as in figure z on page 800.)

### Answers to Self-Checks for Chapter 13

**Page 834:** (1) Most people would think they were positively correlated, but it's possible that they're independent. (2) These must be independent, since there is no possible physical mechanism that could make one have any effect on the other. (3) These cannot be independent, since dying today guarantees that you won't die tomorrow.

**Page 836:** The area under the curve from 130 to 135 cm is about 3/4 of a rectangle. The area from 135 to 140 cm is about 1.5 rectangles. The number of people in the second range is about twice as much. We could have converted these to actual probabilities (1 rectangle = 5 cm  $\times$  0.005 cm<sup>-1</sup> = 0.025), but that would have been pointless because we were just going to compare the two areas.

**Page 841:** On the left-hand side,  $dN$  is a unitless count, and  $dt$  is an infinitesimal amount of time, with units of seconds, so the units are s<sup>-1</sup> as claimed. On the right, both  $N(0)$  and the exponential factor are unitless, so the only units come from the factor of  $1/\tau$ , which again has units of s<sup>-1</sup>.

**Page 850:** The axes of the graph are frequency and photon energy, so its slope is Planck's constant. It doesn't matter if you graph  $e\Delta V$  rather than  $W + e\Delta V$ , because that only changes the y-intercept, not the slope.

**Page 863:** Wavelength is inversely proportional to momentum, so to produce a large wavelength we would need to use electrons with very small momenta and energies. (In practical terms, this isn't very easy to do, since ripping an electron out of an object is a violent process, and it's not so easy to calm the electrons down afterward.)

**Page 872:** Under the ordinary circumstances of life, the accuracy with which we can measure position and momentum of an object doesn't result in a value of  $\Delta p\Delta x$  that is anywhere near the tiny order of magnitude of Planck's constant. We run up against the ordinary limitations on the accuracy of our measuring techniques long before the uncertainty principle becomes an issue.

**Page 876:** No. The equation  $K = p^2/2m$  is nonrelativistic, so it can't be applied to an electron moving at relativistic speeds. Photons always move at relativistic speeds, so it can't be applied to them either.

**Page 878:** Dividing by Planck's constant, a small number, gives a large negative result inside the exponential, so the probability will be very small.

**Page 899:** The original argument was that a kink would have a zero wavelength, which would correspond to an infinite momentum and an infinite kinetic energy, and that would violate conservation of energy. But the kink in this example occurs at  $r = 0$ , which is right on top of the proton, where the electrical energy  $-ke^2/r$  is infinite and *negative*. Since the electrical energy is negative and infinite, we're actually *required* to have an infinite positive kinetic energy in order to come up with a total that conserves energy.

**Page 890:** If you trace a circle going around the center, you run into a series of eight complete wavelengths. Its angular momentum is  $8\hbar$ .

**Page 894:**  $n = 3, \ell = 0, \ell_z = 0$ : one state;  $n = 3, \ell = 1, \ell_z = -1, 0, \text{ or } 1$ : three states;  $n = 3, \ell = 2, \ell_z = -2, -1, 0, 1, \text{ or } 2$ : five states

## Answers

### Answers for Chapter 2

**Page 124, problem 37:**  $K = k_1k_2/(k_1 + k_2) = 1/(1/k_1 + 1/k_2)$

### Answers for Chapter 3

**Page 218, problem 5:** After the collision it is moving at 1/3 of its initial speed, in the same

direction it was initially going (it “follows through”).

**Page 225, problem 41:**  $Q = 1/\sqrt{2}$

**Page 225, problem 43:** (a)  $7 \times 10^{-8}$  radians, or about  $4 \times 10^{-6}$  degrees.

**Page 227, problem 51:** (a)  $R = (2v^2/g) \sin \theta \cos \theta$  (c)  $45^\circ$

**Page 227, problem 52:** (a) The optimal angle is about  $40^\circ$ , and the resulting range is about 124 meters, which is about the length of a home run. (b) It goes about 9 meters farther. For comparison with reality, the stadium’s web site claims a home run goes about 11 meters farther there than in a sea-level stadium.

## Answers for Chapter 5

**Page 339, problem 7:** (c)  $n \approx 16$

**Page 339, problem 9:** (a)  $\sim 2 - 10\%$  (b)  $5\%$  (c) The high end for the body’s actual efficiency is higher than the limit imposed by the laws of thermodynamics. However, the high end of the 1-5 watt range quoted in the problem probably includes large people who aren’t just lying around. Still, it’s impressive that the human body comes so close to the thermodynamic limit.

**Page 340, problem 10:** (a) Looking up the relevant density for air, and converting everything to mks, we get a frequency of 730 Hz. This is on the right order of magnitude, which is promising, considering the crudeness of the approximation. (b) This brings the result down to 400 Hz, which is amazingly close to the observed frequency of 300 Hz.

## Answers for Chapter 6

**Page 382, problem 9:** (b)  $g/2$

**Page 383, problem 13:** Check: The actual length of a flute is about 66 cm from the tip of the mouthpiece to the end of the bell.

**Page 382, problem 8:** (a)  $T = \mu\omega^2 r^2$

**Page 384, problem 17:** (a)  $f = 4\alpha/(1 + \alpha)^2$  (b)  $v_2 = \sqrt{v_1 v_3}$

## Answers for Chapter 10

**Page 642, problem 22:** (a)  $E = 2k\lambda/R$ .

## Answers for Chapter 11

**Page 724, problem 5:** (a)  $I = \lambda v$ .

**Page 725, problem 6:** (b)  $2kI_1 I_2 L/c^2 R$ .

## Answers for Chapter 12

**Page 818, problem 61:**  $f/\epsilon$

## Answers for Chapter 13

**Page 940, problem 40:** about  $10^{-34}$

# Solutions

## Solutions for Chapter 0

**Page 48, problem 6:**

$$134 \text{ mg} \times \frac{10^{-3} \text{ g}}{1 \text{ mg}} \times \frac{10^{-3} \text{ kg}}{1 \text{ g}} = 1.34 \times 10^{-4} \text{ kg}$$

**Page 48, problem 8:** (a) Let's do 10.0 g and 1000 g. The arithmetic mean is 505 grams. It comes out to be 0.505 kg, which is consistent. (b) The geometric mean comes out to be 100 g or 0.1 kg, which is consistent. (c) If we multiply meters by meters, we get square meters. Multiplying grams by grams should give square grams! This sounds strange, but it makes sense. Taking the square root of square grams ( $g^2$ ) gives grams again. (d) No. The superduper mean of two quantities with units of grams wouldn't even be something with units of grams! Related to this shortcoming is the fact that the superduper mean would fail the kind of consistency test carried out in the first two parts of the problem.

**Page 49, problem 12:** (a) They're all defined in terms of the ratio of side of a triangle to another. For instance, the tangent is the length of the opposite side over the length of the adjacent side. Dividing meters by meters gives a unitless result, so the tangent, as well as the other trig functions, is unitless. (b) The tangent function gives a unitless result, so the units on the right-hand side had better cancel out. They do, because the top of the fraction has units of meters squared, and so does the bottom.

**Page 50, problem 17:** The problem requires us to relate  $a$  and  $t$ , for a fixed value of the distance  $\Delta x$ . To find a relationship among these three variables, we start with  $d^2 x/dt^2 = a$ , and integrate twice to find  $\Delta x = \frac{1}{2}at^2$ . This tells us that for a fixed value of  $\Delta x$ , we have  $t \propto 1/\sqrt{a}$ . Decreasing  $a$  by a factor of 3 means that  $t$  will increase by a factor of  $\sqrt{3} = 1.7$ . (The given piece of data, 3, only has one sig fig, but rounding the final result off to one sig fig, giving 2 rather than 1.7, would be a little too severe. As discussed in section 0.1.10, sig figs are only a rule of thumb, and when in doubt, you can change the input data to see how much the output would have changed. The ratio of the gravitational fields on Earth and Mars must be in the range from 2.5 to 3.5, since otherwise the given data would not have been rounded off to 3. Using this range of inputs, the possible range of values for the final result becomes 1.6 to 1.9. The final digit in the 1.7 is therefore a little uncertain, but it's not complete garbage. It carries useful information, and should not be thrown out.)

**Page 51, problem 19:** (a) Solving for  $\Delta x = \frac{1}{2}at^2$  for  $a$ , we find  $a = 2\Delta x/t^2 = 5.51 \text{ m/s}^2$ . (b)  $v = \sqrt{2a\Delta x} = 66.6 \text{ m/s}$ . (c) The actual car's final velocity is less than that of the idealized constant-acceleration car. If the real car and the idealized car covered the quarter mile in the same time but the real car was moving more slowly at the end than the idealized one, the real car must have been going faster than the idealized car at the beginning of the race. The real car apparently has a greater acceleration at the beginning, and less acceleration at the end. This make sense, because every car has some maximum speed, which is the speed beyond which it cannot accelerate.

**Page 52, problem 31:**

$$1 \text{ mm}^2 \times \left( \frac{1 \text{ cm}}{10 \text{ mm}} \right)^2 = 10^{-2} \text{ cm}^2$$

**Page 52, problem 32:** The bigger scope has a diameter that's ten times greater. Area scales as the square of the linear dimensions, so its light-gathering power is a hundred times greater ( $10 \times 10$ ).

**Page 52, problem 33:** Since they differ by two steps on the Richter scale, the energy of the bigger quake is 10000 times greater. The wave forms a hemisphere, and the surface area of the hemisphere over which the energy is spread is proportional to the square of its radius. If the amount of vibration was the same, then the surface areas much be in the ratio of 10000:1, which means that the ratio of the radii is 100:1.

**Page 53, problem 38:** The cone of mixed gin and vermouth is the same shape as the cone of vermouth, but its linear dimensions are doubled, so its volume is 8 times greater. The ratio of gin to vermouth is 7 to 1.

**Page 53, problem 40:** Scaling down the linear dimensions by a factor of 1/10 reduces the volume by a factor of  $(1/10)^3 = 1/1000$ , so if the whole cube is a liter, each small one is one milliliter.

**Page 54, problem 41:** Let's estimate the Great Wall's volume, and then figure out how many bricks that would represent. The wall is famous because it covers pretty much all of China's northern border, so let's say it's 1000 km long. From pictures, it looks like it's about 10 m high and 10 m wide, so the total volume would be  $10^6 \text{ m} \times 10 \text{ m} \times 10 \text{ m} = 10^8 \text{ m}^3$ . If a single brick has a volume of 1 liter, or  $10^{-3} \text{ m}^3$ , then this represents about  $10^{11}$  bricks. If one person can lay 10 bricks in an hour (taking into account all the preparation, etc.), then this would be  $10^{10}$  man-hours.

**Page 54, problem 44:** Directly guessing the number of jelly beans would be like guessing volume directly. That would be a mistake. Instead, we start by estimating the linear dimensions, in units of beans. The contents of the jar look like they're about 10 beans deep. Although the jar is a cylinder, its exact geometrical shape doesn't really matter for the purposes of our order-of-magnitude estimate. Let's pretend it's a rectangular jar. The horizontal dimensions are also something like 10 beans, so it looks like the jar has about  $10 \times 10 \times 10$  or  $\sim 10^3$  beans inside.

### Solutions for Chapter 1

**Page 71, problem 12:** To the person riding the moving bike, bug A is simply going in circles. The only difference between the motions of the two wheels is that one is traveling through space, but motion is relative, so this doesn't have any effect on the bugs. It's equally hard for each of them.

### Solutions for Chapter 2

**Page 118, problem 1:** (a) The energy stored in the gasoline is being changed into heat via frictional heating, and also probably into sound and into energy of water waves. Note that the kinetic energy of the propeller and the boat are not changing, so they are not involved in the energy transformation. (b) The cruising speed would be greater by a factor of the cube root of 2, or about a 26% increase.

**Page 118, problem 2:** We don't have actual masses and velocities to plug in to the equation, but that's OK. We just have to reason in terms of ratios and proportionalities. Kinetic energy is proportional to mass and to the square of velocity, so B's kinetic energy equals  $(13.4 \text{ J})(3.77)/(2.34)^2 = 9.23 \text{ J}$ .

**Page 118, problem 3:** Room temperature is about  $20^\circ\text{C}$ . The fraction of the power that actually goes into heating the water is

$$\frac{(250 \text{ g})/(0.24 \text{ J/g}^\circ\text{C}) \times (100^\circ\text{C}-20^\circ\text{C})/126 \text{ s}}{1.25 \times 10^3 \text{ J/s}} = 0.53$$

So roughly half of the energy is wasted. The wasted energy might be in several forms: heating of the cup, heating of the oven itself, or leakage of microwaves from the oven.

Page 118, problem 5:

$$\begin{aligned}E_{total,i} &= E_{total,f} \\U_i + heat_i &= U_f + heat_f + K_f \\ \frac{1}{2}mv^2 &= U_i - U_f + heat_i - heat_f \\ &= -\Delta U - \Delta heat \\ v &= \sqrt{2 \left( \frac{-\Delta U - \Delta heat}{m} \right)} \\ &= 6.4 \text{ m/s}\end{aligned}$$

**Solutions for Chapter 3**

**Page 218, problem 4:** A conservation law is about addition: it says that when you add up a certain thing, the total always stays the same. Funkosity would violate the additive nature of conservation laws, because a two-kilogram mass would have twice as much funkosity as a pair of one-kilogram masses moving at the same speed.

**Page 219, problem 12:** Momentum is a vector. The total momentum of the molecules is always zero, since the momenta in different directions cancel out on the average. Cooling changes individual molecular momenta, but not the total.

**Page 220, problem 15:**  $a = \Delta v / \Delta t$ , and also  $a = F / m$ , so

$$\begin{aligned}\Delta t &= \frac{\Delta v}{a} \\ &= \frac{m \Delta v}{F} \\ &= \frac{(1000 \text{ kg})(50 \text{ m/s} - 20 \text{ m/s})}{3000 \text{ N}} \\ &= 10 \text{ s}\end{aligned}$$

**Page 221, problem 23:** (a) This is a measure of the box's resistance to a change in its state of motion, so it measures the box's mass. The experiment would come out the same in lunar gravity.

(b) This is a measure of how much gravitational force it feels, so it's a measure of weight. In lunar gravity, the box would make a softer sound when it hit.

(c) As in part a, this is a measure of its resistance to a change in its state of motion: its mass. Gravity isn't involved at all.

**Page 223, problem 34:** (a) The swimmer's acceleration is caused by the water's force on the swimmer, and the swimmer makes a backward force on the water, which accelerates the water backward. (b) The club's normal force on the ball accelerates the ball, and the ball makes a backward normal force on the club, which decelerates the club. (c) The bowstring's normal force accelerates the arrow, and the arrow also makes a backward normal force on the string. This force on the string causes the string to accelerate less rapidly than it would if the bow's force was the only one acting on it. (d) The tracks' backward frictional force slows the locomotive down. The locomotive's forward frictional force causes the whole planet earth to accelerate by a tiny amount, which is too small to measure because the earth's mass is so great.

**Page 224, problem 37:** (a) Spring constants in parallel add, so the spring constant has to be proportional to the cross-sectional area. Two springs in series give half the spring constant, three springs in series give  $1/3$ , and so on, so the spring constant has to be inversely proportional to the length. Summarizing, we have  $k \propto A/L$ .

(b) With the Young's modulus, we have  $k = (A/L)E$ . The spring constant has units of N/m, so the units of  $E$  would have to be  $\text{N/m}^2$ .

**Page 226, problem 44:** By conservation of momentum, the total momenta of the pieces after the explosion is the same as the momentum of the firework before the explosion. However, there is no law of conservation of kinetic energy, only a law of conservation of energy. The chemical energy in the gunpowder is converted into heat and kinetic energy when it explodes. All we can say about the kinetic energy of the pieces is that their total is greater than the kinetic energy before the explosion.

**Page 226, problem 45:** Let  $m$  be the mass of the little puck and  $M = 2.3m$  be the mass of the big one. All we need to do is find the direction of the total momentum vector before the collision, because the total momentum vector is the same after the collision. Given the two components of the momentum vector  $p_x = mv$  and  $p_y = Mv$ , the direction of the vector is  $\tan^{-1}(p_y/p_x) = 23^\circ$  counterclockwise from the big puck's original direction of motion.

**Page 229, problem 62:** We want to find out about the velocity vector  $\mathbf{v}_{BG}$  of the bullet relative to the ground, so we need to add Annie's velocity relative to the ground  $\mathbf{v}_{AG}$  to the bullet's velocity vector  $\mathbf{v}_{BA}$  relative to her. Letting the positive  $x$  axis be east and  $y$  north, we have

$$\begin{aligned}v_{BA,x} &= (140 \text{ mi/hr}) \cos 45^\circ \\ &= 100 \text{ mi/hr} \\ v_{BA,y} &= (140 \text{ mi/hr}) \sin 45^\circ \\ &= 100 \text{ mi/hr}\end{aligned}$$

and

$$\begin{aligned}v_{AG,x} &= 0 \\ v_{AG,y} &= 30 \text{ mi/hr}.\end{aligned}$$

The bullet's velocity relative to the ground therefore has components

$$v_{BG,x} = 100 \text{ mi/hr}$$

and

$$v_{BG,y} = 130 \text{ mi/hr}.$$

Its speed on impact with the animal is the magnitude of this vector

$$\begin{aligned}|\mathbf{v}_{BG}| &= \sqrt{(100 \text{ mi/hr})^2 + (130 \text{ mi/hr})^2} \\ &= 160 \text{ mi/hr}\end{aligned}$$

(rounded off to two significant figures).



**Page 229, problem 63:** Since its velocity vector is constant, it has zero acceleration, and the sum of the force vectors acting on it must be zero. There are three forces acting on the plane: thrust, lift, and gravity. We are given the first two, and if we can find the third we can infer the plane's mass. The sum of the  $y$  components of the forces is zero, so

$$\begin{aligned} 0 &= F_{thrust,y} + F_{lift,y} + F_{g,y} \\ &= |\mathbf{F}_{thrust}| \sin \theta + |\mathbf{F}_{lift}| \cos \theta - mg. \end{aligned}$$

The mass is

$$\begin{aligned} m &= (|\mathbf{F}_{thrust}| \sin \theta + |\mathbf{F}_{lift}| \cos \theta) / g \\ &= 7.0 \times 10^4 \text{ kg}. \end{aligned}$$

**Page 229, problem 64:** (a) Since the wagon has no acceleration, the total forces in both the  $x$  and  $y$  directions must be zero. There are three forces acting on the wagon:  $T$ ,  $\mathbf{F}_g$ , and the normal force from the ground,  $\mathbf{F}_n$ . If we pick a coordinate system with  $x$  being horizontal and  $y$  vertical, then the angles of these forces measured counterclockwise from the  $x$  axis are  $90^\circ - \phi$ ,  $270^\circ$ , and  $90^\circ + \theta$ , respectively. We have

$$\begin{aligned} F_{x,total} &= T \cos(90^\circ - \phi) + F_g \cos(270^\circ) + F_n \cos(90^\circ + \theta) \\ F_{y,total} &= T \sin(90^\circ - \phi) + F_g \sin(270^\circ) + F_n \sin(90^\circ + \theta), \end{aligned}$$

which simplifies to

$$\begin{aligned} 0 &= T \sin \phi - F_n \sin \theta \\ 0 &= T \cos \phi - F_g + F_n \cos \theta. \end{aligned}$$

The normal force is a quantity that we are not given and do not wish to find, so we should choose it to eliminate. Solving the first equation for  $F_n = (\sin \phi / \sin \theta)T$ , we eliminate  $F_n$  from the second equation,

$$0 = T \cos \phi - F_g + T \sin \phi \cos \theta / \sin \theta$$

and solve for  $T$ , finding

$$T = \frac{F_g}{\cos \phi + \sin \phi \cos \theta / \sin \theta}$$

Multiplying both the top and the bottom of the fraction by  $\sin \theta$ , and using the trig identity for  $\sin(\theta + \phi)$  gives the desired result,

$$T = \frac{\sin \theta}{\sin(\theta + \phi)} F_g s$$

(b) The case of  $\phi = 0$ , i.e. pulling straight up on the wagon, results in  $T = F_g$ : we simply support the wagon and it glides up the slope like a chair-lift on a ski slope. In the case of  $\phi = 180^\circ - \theta$ ,  $T$  becomes infinite. Physically this is because we are pulling directly into the ground, so no amount of force will suffice.

**Page 230, problem 65:** (a) If there was no friction, the angle of repose would be zero, so the coefficient of static friction,  $\mu_s$ , will definitely matter. We also make up symbols  $\theta$ ,  $m$  and  $g$  for the angle of the slope, the mass of the object, and the acceleration of gravity. The forces form a triangle just like the one in example 68 on page 203, but instead of a force applied by an external

object, we have static friction, which is less than  $\mu_s F_n$ . As in that example,  $F_s = mg \sin \theta$ , and  $F_s < \mu_s F_n$ , so

$$mg \sin \theta < \mu_s F_n.$$

From the same triangle, we have  $F_n = mg \cos \theta$ , so

$$mg \sin \theta < \mu_s mg \cos \theta.$$

Rearranging,

$$\theta < \tan^{-1} \mu_s.$$

(b) Both  $m$  and  $g$  canceled out, so the angle of repose would be the same on an asteroid.

### Solutions for Chapter 4

**Page 288, problem 1:** The pliers are not moving, so their angular momentum remains constant at zero, and the total torque on them must be zero. Not only that, but each half of the pliers must have zero total torque on it. This tells us that the magnitude of the torque at one end must be the same as that at the other end. The distance from the axis to the nut is about 2.5 cm, and the distance from the axis to the centers of the palm and fingers are about 8 cm. The angles are close enough to  $90^\circ$  that we can pretend they're 90 degrees, considering the rough nature of the other assumptions and measurements. The result is  $(300 \text{ N})(2.5 \text{ cm}) = (F)(8 \text{ cm})$ , or  $F = 90 \text{ N}$ .

**Page 294, problem 37:** The foot of the rod is moving in a circle relative to the center of the rod, with speed  $v = \pi b/T$ , and acceleration  $v^2/(b/2) = (\pi^2/8)g$ . This acceleration is initially upward, and is greater in magnitude than  $g$ , so the foot of the rod will lift off without dragging. We could also worry about whether the foot of the rod would make contact with the floor again before the rod finishes up flat on its back. This is a question that can be settled by graphing, or simply by inspection of figure i on page 276. The key here is that the two parts of the acceleration are both independent of  $m$  and  $b$ , so the result is universal, and it does suffice to check a graph in a single example. In practical terms, this tells us something about how difficult the trick is to do. Because  $\pi^2/8 = 1.23$  isn't much greater than unity, a hit that is just a little too weak (by a factor of  $1.23^{1/2} = 1.11$ ) will cause a fairly obvious qualitative change in the results. This is easily observed if you try it a few times with a pencil.

**Page 296, problem 45:** The moment of inertia is  $I = \int r^2 dm$ . Let the ring have total mass  $M$  and radius  $b$ . The proportionality

$$\frac{M}{2\pi} = \frac{dm}{d\theta}$$

gives a change of variable that results in

$$I = \frac{M}{2\pi} \int_0^{2\pi} r^2 d\theta.$$

If we measure  $\theta$  from the axis of rotation, then  $r = b \sin \theta$ , so this becomes

$$I = \frac{Mb^2}{2\pi} \int_0^{2\pi} \sin^2 \theta d\theta.$$

The integrand averages to  $1/2$  over the  $2\pi$  range of integration, so the integral equals  $\pi$ . We therefore have  $I = \frac{1}{2} Mb^2$ . This is, as claimed, half the value for rotation about the symmetry axis.

## Solutions for Chapter 5

**Page 340, problem 11:** (a) We have

$$\begin{aligned}dP &= \rho g dy \\ \Delta P &= \int \rho g dy,\end{aligned}$$

and since we're taking water to be incompressible, and  $g$  doesn't change very much over 11 km of height, we can treat  $\rho$  and  $g$  as constants and take them outside the integral.

$$\begin{aligned}\Delta P &= \rho g \Delta y \\ &= (1.0 \text{ g/cm}^3)(9.8 \text{ m/s}^2)(11.0 \text{ km}) \\ &= (1.0 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.10 \times 10^4 \text{ m}) \\ &= 1.0 \times 10^8 \text{ Pa} \\ &= 1.0 \times 10^3 \text{ atm.}\end{aligned}$$

The precision of the result is limited to a few percent, due to the compressibility of the water, so we have at most two significant figures. If the change in pressure were exactly a thousand atmospheres, then the pressure at the bottom would be 1001 atmospheres; however, this distinction is not relevant at the level of approximation we're attempting here.

(b) Since the air in the bubble is in thermal contact with the water, it's reasonable to assume that it keeps the same temperature the whole time. The ideal gas law is  $PV = nkT$ , and rewriting this as a proportionality gives

$$V \propto P^{-1},$$

or

$$\frac{V_f}{V_i} = \left(\frac{P_f}{P_i}\right)^{-1} \approx 10^3.$$

Since the volume is proportional to the cube of the linear dimensions, the growth in radius is about a factor of 10.

**Page 340, problem 12:** (a) Roughly speaking, the thermal energy is  $\sim k_B T$  (where  $k_B$  is the Boltzmann constant), and we need this to be on the same order of magnitude as  $ke^2/r$  (where  $k$  is the Coulomb constant). For this type of rough estimate it's not especially crucial to get all the factors of two right, but let's do so anyway. Each proton's average kinetic energy due to motion along a particular axis is  $(1/2)k_B T$ . If two protons are colliding along a certain line in the center-of-mass frame, then their average combined kinetic energy due to motion along that axis is  $2(1/2)k_B T = k_B T$ . So in fact the factors of 2 cancel. We have  $T = ke^2/k_B r$ .

(b) The units are  $\text{K} = (\text{J}\cdot\text{m}/\text{C}^2)(\text{C}^2)/((\text{J}/\text{K})\cdot\text{m})$ , which does work out.

(c) The numerical result is  $\sim 10^{10}$  K, which as suggested is much higher than the temperature at the core of the sun.

**Page 340, problem 13:** If the full-sized brick A undergoes some process, such as heating it with a blowtorch, then we want to be able to apply the equation  $\Delta S = Q/T$  to either the whole brick or half of it, which would be identical to B. When we redefine the boundary of the system to contain only half of the brick, the quantities  $\Delta S$  and  $Q$  are each half as big, because entropy and energy are additive quantities.  $T$ , meanwhile, stays the same, because temperature isn't

additive — two cups of coffee aren't twice as hot as one. These changes to the variables leave the equation consistent, since each side has been divided by 2.

**Page 341, problem 14:** (a) If the expression  $1 + by$  is to make sense, then  $by$  has to be unitless, so  $b$  has units of  $\text{m}^{-1}$ . The input to the exponential function also has to be unitless, so  $k$  also has of  $\text{m}^{-1}$ . The only factor with units on the right-hand side is  $P_o$ , so  $P_o$  must have units of pressure, or Pa.

(b)

$$\begin{aligned} dP &= \rho g dy \\ \rho &= \frac{1}{g} \frac{dP}{dy} \\ &= \frac{P_o}{g} e^{-ky} (-k - kby + b) \end{aligned}$$

(c) The three terms inside the parentheses on the right all have units of  $\text{m}^{-1}$ , so it makes sense to add them, and the factor in parentheses has those units. The units of the result from b then look like

$$\begin{aligned} \frac{\text{kg}}{\text{m}^3} &= \frac{\text{Pa}}{\text{m/s}^2} \text{m}^{-1} \\ &= \frac{\text{N/m}^2}{\text{m}^2/\text{s}^2} \\ &= \frac{\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}}{\text{m}^2/\text{s}^2}, \end{aligned}$$

which checks out.

## Solutions for Chapter 7

**Page 447, problem 17:** (a) Plugging in, we find

$$\sqrt{\frac{1-w}{1+w}} = \sqrt{\frac{1-u}{1+u}} \sqrt{\frac{1-v}{1+v}}.$$

(b) First let's simplify by squaring both sides.

$$\frac{1-w}{1+w} = \frac{1-u}{1+u} \cdot \frac{1-v}{1+v}.$$

For convenience, let's write  $A$  for the right-hand side of this equation. We then have

$$\begin{aligned} \frac{1-w}{1+w} &= A \\ 1-w &= A + Aw. \end{aligned}$$

Solving for  $w$ ,

$$\begin{aligned} w &= \frac{1-A}{1+A} \\ &= \frac{(1+u)(1+v) - (1-u)(1-v)}{(1+u)(1+v) + (1-u)(1-v)} \\ &= \frac{2(u+v)}{2(1+uv)} \\ &= \frac{u+v}{1+uv} \end{aligned}$$

(c) This is all in units where  $c = 1$ . The correspondence principle says that we should get  $w \approx u + v$  when both  $u$  and  $v$  are small compared to 1. Under those circumstances,  $uv$  is the product of two very small numbers, which makes it very, very small. Neglecting this term in the denominator, we recover the nonrelativistic result.

**Page 447, problem 18:** Among the spacelike vectors,  $\mathbf{a}$  and  $\mathbf{e}$  are clearly congruent, because they're the same except for a rotation in space; this is the same as the definition of congruence in ordinary Euclidean geometry, where rotation doesn't matter. Vector  $\mathbf{b}$  is also congruent to these, since it represents an interval  $3^2 - 5^2 = -4^2$ , just like the other two.

The lightlike vectors  $\mathbf{c}$  and  $\mathbf{d}$  both represent intervals of zero, so they're congruent, even though  $\mathbf{c}$  is a double-scale version of  $\mathbf{d}$ .

The timelike vectors  $\mathbf{f}$  and  $\mathbf{g}$  are not congruent to each other or to any of the others;  $\mathbf{f}$  represents an interval of  $2^2$ , while  $\mathbf{g}$ 's interval is  $4^2$ .

**Page 448, problem 22:** At the center of each of the large triangle's sides, the angles add up to  $180^\circ$  because they form a straight line. Therefore  $4s = S + 3 \times 180^\circ$ , so  $S - 180^\circ = 4(s - 180^\circ)$ .

**Page 449, problem 28:** By the equivalence principle, we can adopt a frame tied to the tossed clock, B, and in this frame there is no gravitational field. We see a desk and clock A go by. The desk applies a force to clock A, decelerating it and then reaccelerating it so that it comes back. We've already established that the effect of motion is to slow down time, so clock A reads a smaller time interval.

### Hints for Chapter 8

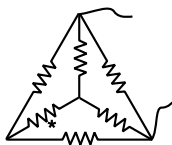
**Page 512, problem 15:** The force on the lithium ion is the vector sum of all the forces of all the quadrillions of sodium and chlorine atoms, which would obviously be too laborious to calculate. Nearly all of these forces, however, are canceled by a force from an ion on the opposite side of the lithium.

### Solutions for Chapter 9

**Page 550, problem 1:**  $\Delta t = \Delta q/I = e/I = 0.16 \mu s$

**Page 551, problem 12:** In series, they give  $11 \text{ k}\Omega$ . In parallel, they give  $(1/1 \text{ k}\Omega + 1/10 \text{ k}\Omega)^{-1} = 0.9 \text{ k}\Omega$ .

**Page 554, problem 25:** The actual shape is irrelevant; all we care about is what's connected to what. Therefore, we can draw the circuit flattened into a plane. Every vertex of the tetrahedron is adjacent to every other vertex, so any two vertices to which we connect will give the same resistance. Picking two arbitrarily, we have this:



This is unfortunately a circuit that cannot be converted into parallel and series parts, and that's what makes this a hard problem! However, we can recognize that by symmetry, there is zero current in the resistor marked with an asterisk. Eliminating this one, we recognize the whole arrangement as a triple parallel circuit consisting of resistances  $R$ ,  $2R$ , and  $2R$ . The resulting resistance is  $R/2$ .

**Page 555, problem 29:** (a) Conservation of energy gives

$$\begin{aligned}U_A &= U_B + K_B \\K_B &= U_A - U_B \\ \frac{1}{2}mv^2 &= e\Delta V \\ v &= \sqrt{\frac{2e\Delta V}{m}}\end{aligned}$$

(b) Plugging in numbers, we get  $5.9 \times 10^7$  m/s. This is about 20% of the speed of light, so the nonrelativistic assumption was good to at least a rough approximation.

**Page 556, problem 32:** It's much more practical to measure voltage differences. To measure a current, you have to break the circuit somewhere and insert the meter there, but it's not possible to disconnect the circuits sealed inside the board.

### Solutions for Chapter 10

**Page 640, problem 16:** By symmetry, the field is always directly toward or away from the center. We can therefore calculate it along the  $x$  axis, where  $r = x$ , and the result will be valid for any location at that distance from the center.

$$\begin{aligned}E &= -\frac{dV}{dx} \\ &= -\frac{d}{dx}(x^{-1}e^{-x}) \\ &= x^{-2}e^{-x} + x^{-1}e^{-x}\end{aligned}$$

At small  $x$ , near the proton, the first term dominates, and the exponential is essentially 1, so we have  $E \propto x^{-2}$ , as we expect from the Coulomb force law. At large  $x$ , the second term dominates, and the field approaches zero faster than an exponential.

**Page 648, problem 56:**

$$\begin{aligned}\sin(a+b) &= (e^{i(a+b)} - e^{-i(a+b)})/2i \\ &= (e^{ia}e^{ib} - e^{-ia}e^{-ib})/2i \\ &= [(\cos a + i \sin a)(\cos b + i \sin b) - (\cos a - i \sin a)(\cos b - i \sin b)]/2i \\ &= \cos a \sin b + \sin a \cos b\end{aligned}$$

By a similar computation, we find  $\cos(a+b) = \cos a \cos b - \sin a \sin b$ .

**Page 648, problem 57:** If  $z^3 = 1$ , then we know that  $|z| = 1$ , since cubing  $z$  cubes its magnitude. Cubing  $z$  triples its argument, so the argument of  $z$  must be a number that, when tripled, is equivalent to an angle of zero. There are three possibilities:  $0 \times 3 = 0$ ,  $(2\pi/3) \times 3 = 2\pi$ , and  $(4\pi/3) \times 3 = 4\pi$ . (Other possibilities, such as  $(32\pi/3)$ , are equivalent to one of these.) The solutions are:

$$z = 1, e^{2\pi i/3}, e^{4\pi i/3}$$

**Page 648, problem 58:** We can think of this as a polynomial in  $x$  or a polynomial in  $y$  — their roles are symmetric. Let's call  $x$  the variable. By the fundamental theorem of algebra, it

must be possible to factor it into a product of three linear factors, if the coefficients are allowed to be complex. Each of these factors causes the product to be zero for a certain value of  $x$ . But the condition for the expression to be zero is  $x^3 = y^3$ , which basically means that the ratio of  $x$  to  $y$  must be a third root of 1. The problem, then, boils down to finding the three third roots of 1, as in problem 57. Using the result of that problem, we find that there are zeroes when  $x/y$  equals 1,  $e^{2\pi i/3}$ , and  $e^{4\pi i/3}$ . This tells us that the factorization is  $(x - y)(x - e^{2\pi i/3}y)(x - e^{4\pi i/3}y)$ .

The second part of the problem asks us to factorize as much as possible using real coefficients. Our only hope of doing this is to multiply out the two factors that involve complex coefficients, and see if they produce something real. In fact, we can anticipate that it will work, because the coefficients are complex conjugates of one another, and when a quadratic has two complex roots, they are conjugates. The result is  $(x - y)(x^2 + xy + y^2)$ .

### Solutions for Chapter 11

**Page 735, problem 51:** (a) For a material object,  $\mathbf{p} = m\mathbf{v}$ . The velocity vector reverses itself, but mass is still positive, so the momentum vector is reversed.

(b) In the forward-time universe, conservation of momentum is  $\mathbf{p}_{1,i} + \mathbf{p}_{2,i} = \mathbf{p}_{1,f} + \mathbf{p}_{2,f}$ . In the backward-time universe, all the momenta are reversed, but that just negates both sides of the equation, so momentum is still conserved.

**Page 737, problem 54:** Note that in the Biot-Savart law, the variable  $\mathbf{r}$  is defined as a vector that points from the current to the point at which the field is being calculated, whereas in the polar coordinates used to express the equation of the spiral, the vector more naturally points the opposite way. This requires some fiddling with signs, which I'll suppress, and simply identify  $d\ell$  with  $d\mathbf{r}$ .

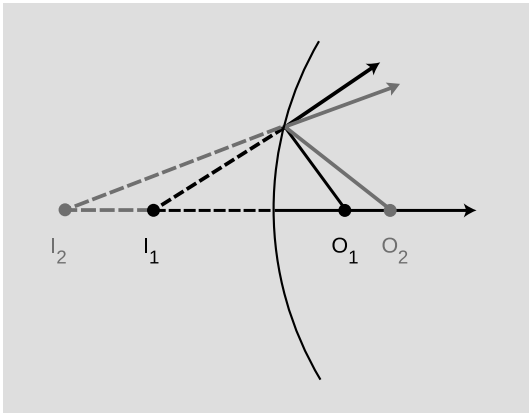
$$\mathbf{B} = \frac{kI}{c^2} \int \frac{d\ell \times \mathbf{r}}{r^3}$$

The vector  $d\mathbf{r}$  has components  $dx = w(\cos\theta - \theta \sin\theta)$  and  $dy = w(\sin\theta + \theta \cos\theta)$ . Evaluating the vector cross product, and substituting  $\theta/w$  for  $r$ , we find

$$\begin{aligned} \mathbf{B} &= \frac{kI}{c^2 w} \int \frac{\theta(\cos\theta \sin\theta - \theta \sin^2\theta - \cos\theta \sin\theta - \theta \cos^2\theta) d\theta}{\theta^3} \\ &= \frac{kI}{c^2 w} \int \frac{d\theta}{\theta} \\ &= \frac{kI}{c^2 w} \ln \frac{\theta_2}{\theta_1} \\ &= \frac{kI}{c^2 w} \ln \frac{b}{a} \end{aligned}$$

### Solutions for Chapter 12

**Page 806, problem 16:** See the ray diagram below. Decreasing  $\theta_o$  decreases  $\theta_i$ , so the equation  $\theta_f = \pm\theta_i + \pm\theta_o$  must have opposite signs on the right. Since  $\theta_o$  is bigger than  $\theta_i$ , the only way to get a positive  $\theta_f$  is if the signs are  $\theta_f = -\theta_i + \theta_o$ . This gives  $1/f = -1/d_i + 1/d_o$ .



**Page 806, problem 19:** (a) The object distance is less than the focal length, so the image is virtual: because the object is so close, the cone of rays is diverging too strongly for the mirror to bring it back to a focus. (b) At an object distance of 30 cm, it's clearly going to be real. With the object distance of 20 cm, we're right at the crossing-point between real and virtual. For this object position, the reflected rays will be parallel. We could consider this to be an image at infinity. (c),(d) A diverging mirror can only make virtual images.

**Page 810, problem 39:** Since  $d_o$  is much greater than  $d_i$ , the lens-film distance  $d_i$  is essentially the same as  $f$ . (a) Splitting the triangle inside the camera into two right triangles, straightforward trigonometry gives

$$\theta = 2 \tan^{-1} \frac{w}{2f}$$

for the field of view. This comes out to be  $39^\circ$  and  $64^\circ$  for the two lenses. (b) For small angles, the tangent is approximately the same as the angle itself, provided we measure everything in radians. The equation above then simplifies to

$$\theta = \frac{w}{f}$$

The results for the two lenses are  $.70 \text{ rad} = 40^\circ$ , and  $1.25 \text{ rad} = 72^\circ$ . This is a decent approximation.

(c) With the 28-mm lens, which is closer to the film, the entire field of view we had with the 50-mm lens is now confined to a small part of the film. Using our small-angle approximation  $\theta = w/f$ , the amount of light contained within the same angular width  $\theta$  is now striking a piece of the film whose linear dimensions are smaller by the ratio  $28/50$ . Area depends on the square of the linear dimensions, so all other things being equal, the film would now be overexposed by a factor of  $(50/28)^2 = 3.2$ . To compensate, we need to shorten the exposure by a factor of 3.2.

**Page 818, problem 59:** One surface is curved outward and one inward. Therefore the minus sign applies in the lensmaker's equation. Since the radii of curvature are equal, the quantity  $1/r_1 - 1/r_2$  equals zero, and the resulting focal length is infinite. A big focal length indicates a weak lens. An infinite focal length tells us that the lens is infinitely weak — it doesn't focus or defocus rays at all.

### Solutions for Chapter 13

**Page 942, problem 48:** The expressions  $|\Psi|^2$  and  $|\Psi^2|$  are identical, because the magnitude of a product is the product of the magnitudes. These expressions give positive real numbers as their



results, which makes sense for a probability density. The expression  $\Psi^2$  need not be real, and if it is real, it may be negative. It cannot be interpreted as a probability density. As a concrete example, suppose that  $\Psi = bi$ , where  $b$  is a real number with units. Then  $|\Psi|^2 = |\Psi^2| = b^2$ , which is real and positive, but  $\Psi^2 = -b^2$ , which clearly can't be interpreted probabilistically, because it's negative.

**Page 942, problem 49:** (a) The quantity  $x - y$  vanishes along the line  $y = x$  lying in the first quadrant at a 45-degree angle between the axes. Squaring produces a trough parallel to this line, with a parabolic cross-section. Geometrically, the Laplacian can be interpreted as a measure of how much the value of  $f$  at a point differs from its average value on a small circle centered on that point. The trough is concave up, so we can predict that the Laplacian will be positive everywhere.

(b) The zero result is clearly wrong because it disagrees with our conclusion from part a that the Laplacian is positive. A correct calculation gives  $\partial^2(x - y)^2/\partial x^2 + \partial^2(x - y)^2/\partial y^2 = 4$ .

(c) If we rotate our coordinate axes counterclockwise by 45 degrees, then we have a parabolic trough oriented along the  $x$  axis. In terms of these new coordinates,  $\partial f/\partial x = 0$ , while  $\partial f/\partial y$  is nonzero almost everywhere.

Remark: The mistake described in the question is a common one, and is apparently based on the idea that the notation  $\nabla^2$  must mean applying an operator  $\nabla$  twice. For those with some exposure to vector calculus, it may be of interest to note that the Laplacian *is* equivalent to the divergence of the gradient, which can be notated either  $\text{div}(\text{grad } f)$  or  $\nabla \cdot (\nabla f)$ . The important thing to recognize is that the gradient, notated  $\text{grad } f$  or  $\nabla f$ , outputs a *vector*, not a scalar like the quantity  $Q$  defined in this problem.

# Appendix 5: Useful Data

## 0.6 Notation and terminology, compared with other books

Almost all the notation and terminology in *Simple Nature* is standard, but there are some cases where there is no universal standard, and a very few cases where I've intentionally deviated from a universal standard. The notation used by physicists is also different from that used by electrical and mechanical engineers; I use physics terminology and notation (notably  $\sqrt{-1} = i$ , not  $j$ , and “torque” rather than “moment”), but employ the SI system of units used in engineering, rather than the cgs units favored by some physicists.

Nonstandard terminology:

**Potential energy** is referred to in this book as *interaction energy*, or according to its type: *gravitational energy*, *electrical energy*, etc.

**The potential**, in an electrical context, is referred to as *voltage*, e.g. I say that  $V = kq/r$  is the voltage surrounding a point charge.

**Heat and thermal energy** are both referred to as *heat*. This is in keeping with casual usage among scientists, but formal written usage dictates the use of “thermal energy” to mean the kinetic energy an object has because of its molecules' random motion, while “heat” is the transfer of thermal energy.

Notation for which there is no universal standard:

**Kinetic energy** is written  $K$ . Standard notation is  $K$ ,  $T$ , or  $KE$ .

**Interaction energy** is written  $U$ . Standard notation is  $U$ ,  $V$ , or  $PE$ .

**The unit vectors** are  $\hat{x}, \hat{y}, \hat{z}$ . Standard notation is either  $\hat{x}, \hat{y}, \hat{z}$  or  $\hat{i}, \hat{j}, \hat{k}$ .

**Distance from an axis** in cylindrical coordinates is  $R$ . A more common notation in math books is  $\rho$ , but this would conflict with the standard physics notation for the charge density.

**Vibrations** do not have very well standardized terminology or notation. I use “frequency” to refer to both  $f$  and  $\omega$ , depending on the context to make it clear which is meant. The frequency of free, damped oscillations is  $\omega_f$ , which is only approximately the same as  $\omega_o = \sqrt{k/m}$ . The full width at half-maximum of the resonance peak (on a plot of energy versus frequency) is  $\Delta\omega$ .

**The coupling constants** for electricity and magnetism are written as  $k$  and  $k/c^2$ . This is standard notation, but it would be more common in SI calculations to see everything expressed in terms of  $\epsilon_o = 1/4\pi k$  and  $\mu_o = 4\pi k/c^2$ . Numerically, we have  $k = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$  and  $k/c^2 = 10^{-7} \text{ N}/\text{A}^2$ , the latter being an exact relation.

## .0.7 Notation and units

quantity	unit	symbol
distance	meter, m	$x, \Delta x$
time	second, s	$t, \Delta t$
mass	kilogram, kg	$m$
density	$\text{kg}/\text{m}^3$	$\rho$
force	newton, $1 \text{ N} = 1 \text{ kg} \cdot \text{m}/\text{s}^2$	$F$
velocity	m/s	$v$
acceleration	$\text{m}/\text{s}^2$	$a$
gravitational field	$\text{J}/\text{kg} \cdot \text{m}$ or $\text{m}/\text{s}^2$	$g$
energy	joule, J	$E$ (also electric field)
momentum	$\text{kg} \cdot \text{m}/\text{s}$	$p$
angular momentum	$\text{kg} \cdot \text{m}^2/\text{s}$ or $\text{J} \cdot \text{s}$	$L$ (also inductance)
power	watt, $1 \text{ W} = 1 \text{ J}/\text{s}$	$P$ (also pressure)
pressure	$1 \text{ Pa} = 1 \text{ N}/\text{m}^2$	$P$ (also power)
temperature	K	$T$ (also period)
period	s	$T$ (also temperature)
wavelength	m	$\lambda$
frequency	$\text{s}^{-1}$ or Hz	$f$
charge	coulomb, C	$q$
voltage	volt, $1 \text{ V} = 1 \text{ J}/\text{C}$	$V$
current	ampere, $1 \text{ A} = 1 \text{ C}/\text{s}$	$I$
resistance	ohm, $1 \Omega = 1 \text{ V}/\text{A}$	$R$
capacitance	farad, $1 \text{ F} = 1 \text{ C}/\text{V}$	$C$
inductance	henry, $1 \text{ H} = 1 \text{ V} \cdot \text{s}/\text{A}$	$L$ (also angular momentum)
electric field	$\text{V}/\text{m}$ or $\text{N}/\text{C}$	$E$ (also energy)
magnetic field	tesla, $1 \text{ T} = 1 \text{ N} \cdot \text{s}/\text{C} \cdot \text{m}$	$B$
focal length	m	$f$
magnification	unitless	$M$
index of refraction	unitless	$n$
electron wavefunction	$\text{m}^{-3/2}$	$\Psi$

## .0.8 Fundamental constants

gravitational constant	$G = 6.67 \times 10^{-11} \text{ J} \cdot \text{m}/\text{kg}^2$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J}/\text{K}$
Coulomb constant	$k = 8.99 \times 10^9 \text{ J} \cdot \text{m}/\text{C}^2$ or $\text{N} \cdot \text{m}^2/\text{C}^2$
quantum of charge	$e = 1.60 \times 10^{-19} \text{ C}$
speed of light	$c = 3.00 \times 10^8 \text{ m}/\text{s}$
Planck's constant	$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

Note the use of the same notation,  $k$ , for both the Boltzmann constant and the Coulomb constant.

### .0.9 Metric prefixes

M-	mega-	$10^6$
k-	kilo-	$10^3$
m-	milli-	$10^{-3}$
$\mu$ - (Greek mu)	micro-	$10^{-6}$
n-	nano-	$10^{-9}$
p-	pico-	$10^{-12}$
f-	femto-	$10^{-15}$

Note that the exponents go in steps of three. The exception is centi-,  $10^{-2}$ , which is used only in the centimeter, and this doesn't require memorization, because a cent is  $10^{-2}$  dollars.

### .0.10 Nonmetric units

Nonmetric units in terms of metric ones:

1 inch	= 25.4 mm (by definition)
1 pound (lb)	= 4.5 newtons of force
1 scientific calorie	= 4.18 J
1 nutritional calorie	= $4.18 \times 10^3$ J
1 gallon	= $3.78 \times 10^3$ cm <sup>3</sup>
1 horsepower	= 746 W

The pound is a unit of force, so it converts to newtons, not kilograms. A one-kilogram mass at the earth's surface experiences a gravitational force of  $(1 \text{ kg})(9.8 \text{ m/s}^2) = 9.8 \text{ N} = 2.2 \text{ lb}$ . The nutritional information on food packaging typically gives energies in units of calories, but those so-called calories are really kilocalories.

Relationships among U.S. units:

1 foot (ft)	= 12 inches
1 yard (yd)	= 3 feet
1 mile (mi)	= 5280 feet
1 ounce (oz)	= 1/16 pound

### .0.11 The Greek alphabet

$\alpha$	A	alpha	$\iota$	I	iota	$\rho$	P	rho
$\beta$	B	beta	$\kappa$	K	kappa	$\sigma$	$\Sigma$	sigma
$\gamma$	$\Gamma$	gamma	$\lambda$	$\Lambda$	lambda	$\tau$	T	tau
$\delta$	$\Delta$	delta	$\mu$	M	mu	$\upsilon$	Y	upsilon
$\epsilon$	E	epsilon	$\nu$	N	nu	$\phi$	$\Phi$	phi
$\zeta$	Z	zeta	$\xi$	$\Xi$	xi	$\chi$	X	chi
$\eta$	H	eta	$\omicron$	O	omicron	$\psi$	$\Psi$	psi
$\theta$	$\Theta$	theta	$\pi$	$\Pi$	pi	$\omega$	$\Omega$	omega

### .0.12 Subatomic particles

particle	mass (kg)	charge	radius (fm)
electron	$9.109 \times 10^{-31}$	$-e$	$\lesssim 0.01$
proton	$1.673 \times 10^{-27}$	$+e$	$\sim 1.1$
neutron	$1.675 \times 10^{-27}$	0	$\sim 1.1$
neutrino	$\sim 10^{-39}$ kg ?	0	?

The radii of protons and neutrons can only be given approximately, since they have fuzzy

surfaces. For comparison, a typical atom is about a million fm in radius.

### .0.13 Earth, moon, and sun

body	mass (kg)	radius (km)	radius of orbit (km)
earth	$5.97 \times 10^{24}$	$6.4 \times 10^3$	$1.49 \times 10^8$
moon	$7.35 \times 10^{22}$	$1.7 \times 10^3$	$3.84 \times 10^5$
sun	$1.99 \times 10^{30}$	$7.0 \times 10^5$	—

### .0.14 The periodic table

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	* 72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	** 104 Rf	105 Ha	106	107	108	109	110	111	112	113	114	115	116	117	118
			* 58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
			** 90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

### .0.15 Atomic masses

These atomic masses are given in atomic mass units (u), where by definition the mass of an atom of the isotope carbon-12 equals 12 u. One atomic mass unit is the same as about  $1.66 \times 10^{-27}$  kg. Data are only given for naturally occurring elements.

Ag	107.9	Eu	152.0	Mo	95.9	Sc	45.0
Al	27.0	F	19.0	N	14.0	Se	79.0
Ar	39.9	Fe	55.8	Na	23.0	Si	28.1
As	74.9	Ga	69.7	Nb	92.9	Sn	118.7
Au	197.0	Gd	157.2	Nd	144.2	Sr	87.6
B	10.8	Ge	72.6	Ne	20.2	Ta	180.9
Ba	137.3	H	1.0	Ni	58.7	Tb	158.9
Be	9.0	He	4.0	O	16.0	Te	127.6
Bi	209.0	Hf	178.5	Os	190.2	Ti	47.9
Br	79.9	Hg	200.6	P	31.0	Tl	204.4
C	12.0	Ho	164.9	Pb	207.2	Tm	168.9
Ca	40.1	In	114.8	Pd	106.4	U	238
Ce	140.1	Ir	192.2	Pt	195.1	V	50.9
Cl	35.5	K	39.1	Pr	140.9	W	183.8
Co	58.9	Kr	83.8	Rb	85.5	Xe	131.3
Cr	52.0	La	138.9	Re	186.2	Y	88.9
Cs	132.9	Li	6.9	Rh	102.9	Yb	173.0
Cu	63.5	Lu	175.0	Ru	101.1	Zn	65.4
Dy	162.5	Mg	24.3	S	32.1	Zr	91.2
Er	167.3	Mn	54.9	Sb	121.8		

# Appendix 6: Summary

Notation and units are summarized on page 985.

## Chapter 0, Introduction and Review, page 13

Physics is the use of the scientific method to study the behavior of light and matter. The scientific method requires a cycle of theory and experiment, theories with both predictive and explanatory value, and reproducible experiments.

The metric system is a simple, consistent framework for measurement built out of the meter, the kilogram, and the second plus a set of prefixes denoting powers of ten. The most systematic method for doing conversions is shown in the following example:

$$370 \text{ ms} \times \frac{10^{-3} \text{ s}}{1 \text{ ms}} = 0.37 \text{ s}$$

Mass is a measure of the amount of a substance. Mass can be defined gravitationally, by comparing an object to a standard mass on a double-pan balance, or in terms of inertia, by comparing the effect of a force on an object to the effect of the same force on a standard mass. The two definitions are found experimentally to be proportional to each other to a high degree of precision, so we usually refer simply to “mass,” without bothering to specify which type.

A force is that which can change the motion of an object. The metric unit of force is the Newton, defined as the force required to accelerate a standard 1-kg mass from rest to a speed of 1 m/s in 1 s.

Scientific notation means, for example, writing  $3.2 \times 10^5$  rather than 320000.

Writing numbers with the correct number of significant figures correctly communicates how accurate they are. As a rule of thumb, the final result of a calculation is no more accurate than, and should have no more significant figures than, the least accurate piece of data.

Nature behaves differently on large and small scales. Galileo showed that this results fundamentally from the way area and volume scale. Area scales as the second power of length,  $A \propto L^2$ , while volume scales as length to the third power,  $V \propto L^3$ .

An order of magnitude estimate is one in which we do not attempt or expect an exact answer. The main reason why the uninitiated have trouble with order-of-magnitude estimates is that the human brain does not intuitively make accurate estimates of area and volume. Estimates of area and volume should be approached by first estimating linear dimensions, which one’s brain has a feel for.

Velocity,  $dx/dt$ , measures how fast an object is moving. Acceleration,  $d^2x/dt^2$ , measures how quickly its velocity is changing. For motion with constant acceleration, we have these useful

relations:

$$a = \frac{\Delta v}{\Delta t}$$
$$x = \frac{1}{2}at^2 + v_0t + x_0$$
$$v_f^2 = v_0^2 + 2a\Delta x$$

## Chapter 1, Conservation of Mass, page 55

*Conservation laws* are the foundation of physics. A conservation law states that a certain quantity can be neither created nor destroyed; the total amount of it remains the same.

*Mass* is a conserved quantity in classical physics, i.e. physics before Einstein. This is plausible, since we know that matter is composed of subatomic particles; if the particles are neither created or destroyed, then it makes sense that the total mass will remain the same. There are two ways of defining mass.

*Gravitational mass* is defined by measuring the effect of gravity on a particular object, and comparing with some standard object, taking care to test both objects at a location where the strength of gravity is the same.

*Inertial mass* is defined by measuring how much a particular object resists a change in its state of motion. For instance, an object placed on the end of a spring will oscillate if the spring is initially compressed, and a more massive object will take longer to complete one oscillation.

*Inertial and gravitational mass are equivalent:* experiments show to a very high degree of precision that any two objects with the same inertial mass have the same gravitational mass as well.

The definition of inertial mass depends on a correct but counterintuitive assumption: that an object resists a change in its state of motion. Most people intuitively believe that motion has a natural tendency to slow down. This cannot be correct as a general statement, because “to slow down” is not a well-defined concept unless we specify what we are measuring motion relative to. This insight is credited to Galileo, and the general principle of *Galilean relativity* states that the laws of physics are the same in all inertial frames of reference. In other words, there is no way to distinguish a moving frame of reference from one that is at rest. To establish which frames of reference are inertial, we first must find one inertial frame in which objects appear to obey Galilean relativity. The surface of the earth is an inertial frame to a reasonably good approximation, and the frame of reference of the stars is an even better one. Once we have found one inertial frame of reference, any other frame is inertial which is moving in a straight line at constant velocity relative to the first one. For instance, if the surface of the earth is an approximately inertial frame, then a train traveling in a straight line at constant speed is also approximately an inertial frame.

The unit of mass is the kilogram, which, along with the meter and the second, forms the basis for the SI system of units (also known as the mks system). A fundamental skill in science is to know the definitions of the most common metric prefixes, which are summarized on page 986, and to be able to convert among them.

One consequence of Einstein’s theory of special relativity is that *mass can be converted to energy and energy to mass*. This prediction has been verified amply by experiment. Thus the conserved quantity is not really mass but rather the total “mass-energy,”  $m + E/c^2$ , where  $c$  is the speed of light. Since the speed of light is a large number, the  $E/c^2$  term is ordinarily small

in everyday life, which is why we can usually neglect it.

## Chapter 2, Conservation of Energy, page 73

We observe that certain processes are physically impossible. For example, there is no process that can heat up an object without using up fuel or having some other side effect such as cooling a different object. We find that we can neatly separate the possible processes from the impossible by defining a single numerical quantity, called *energy*, which is conserved. Energy comes in many forms, such as heat, motion, sound, light, the energy required to melt a solid, and gravitational energy (e.g. the energy that depends on the distance between a rock and the earth). Because it has so many forms, we can arbitrarily choose one form, heat, in order to define a standard unit for our numerical scale of energy. Energy is measured in units of joules (J), and one joule can be defined as the amount of energy required in order to raise the temperature of a certain amount of water by a certain number of degrees. (The numbers are not worth memorizing.) *Power* is defined as the rate of change of energy  $P = dE/dt$ , and the unit of power is the watt,  $1 \text{ W} = 1 \text{ J/s}$ .

Once we have defined one type of energy numerically, we can perform experiments that establish the mathematical rules governing other types of energy. For example, in his paddlewheel experiment, James Joule allowed weights to drop through a certain height and spin paddlewheels inside sealed canisters of water, thereby heating the water through friction. Since in this book we define the joule unit in terms of the temperature of water, we can think of the paddlewheel experiment as establishing a rule for the *gravitational energy* of a mass which is at a certain height,

$$dU_g = mg dy,$$

where  $dU_g$  is the infinitesimal change in the gravitational energy of a mass  $m$  when its height is changed by an infinitesimal amount  $dy$  in the vertical direction. The quantity  $g$  is called the *gravitational field*, and at the earth's surface it has a numerical value of about  $10 \text{ J/kg}\cdot\text{m}$ . That is, about 10 joules of energy are required in order to raise a one-kilogram mass by one meter. (The gravitational field  $g$  also has the interpretation that when we drop an object, its acceleration,  $d^2 y/dt^2$ , is equal to  $g$ .)

Using similar techniques, we find that the energy of a moving object, called its *kinetic energy*, is given by

$$K = \frac{1}{2}mv^2,$$

where  $m$  is its mass and  $v$  its velocity. The proportionality factor equals  $1/2$  exactly by the design of the SI system of units, and since the SI is based on the meter, the kilogram, and the second, the joule is considered to be a derived unit,  $1 \text{ J} = 1 \text{ kg}\cdot\text{m}^2/\text{s}^2$ .

When the interaction energy  $U$  has a local maximum or minimum with respect to the position of an object ( $dU/dx = 0$ ), then the object is in *equilibrium* at that position. For example, if a weight is hanging from a rope, and is initially at rest at the bottom, then it must remain at rest, because this is a position of minimum gravitational energy  $U_g$ ; to move, it would have to increase both its kinetic and its gravitational energy, which would violate conservation of energy, since the total energy would increase.

Since kinetic energy is independent of the direction of motion, conservation of energy is often insufficient to predict the direction of an object's motion. However, many of the physically impossible motions can be ruled out by the trick of imposing conservation of energy in some other frame of reference. By this device, we can solve the important problem of *projectile motion*: even if the projectile has horizontal motion, we can imagine ourselves in a frame of reference in which



we are moving along with the projectile horizontally. In this frame of reference, the projectile has no horizontal motion, and its vertical motion has constant acceleration  $g$ . Switching back to a frame of reference in which its horizontal velocity is not zero, we find that a projectile's horizontal and vertical motions are independent, and that the horizontal motion is at constant velocity.

Even in one-dimensional motion, it is seldom possible to solve real-world problems and predict the motion of an object in closed form. However, there are straightforward numerical techniques for solving such problems.

From observations of the motion of the planets, we infer that the *gravitational interaction between any two objects* is given by  $U_g = -Gm_1m_2/r$ , where  $r$  is the distance between them. When the sizes of the objects are not small compared to their separation, the definition of  $r$  becomes vague; for this reason, we should interpret this fundamentally as the law governing the gravitational interactions between individual atoms. However, in the special case of a spherically symmetric mass distribution, there is a shortcut: the *shell theorem* states that the gravitational interaction between a spherically symmetric shell of mass and a particle on the outside of the shell is the same as if the shell's mass had all been concentrated at its center. An astronomical body like the earth can be broken down into concentric shells of mass, and so its gravitational interactions with external objects can also be calculated simply by using the center-to-center distance.

Energy appears to come in a bewildering variety of forms, but matter is made of atoms, and thus if we restrict ourselves to the study of mechanical systems (containing material objects, not light), all the forms of energy we observe must be explainable in terms of the behavior and interactions of atoms. Indeed, at the atomic level the picture is much simpler. Fundamentally, all the familiar forms of mechanical energy arise from either the kinetic energy of atoms or the energy they have because they interact with each other via gravitational or electrical forces. For example, when we stretch a spring, we distort the latticework of atoms in the metal, and this change in the interatomic distances involves an increase in the atoms' electrical energies.

An equilibrium is a local minimum of  $U(x)$ , and up close, any minimum looks like a parabola. Therefore, small oscillations around an equilibrium exhibit universal behavior, which depends only on the object's mass,  $m$ , and on the tightness of curvature of the minimum, parametrized by the quantity  $k = d^2U/dx^2$ . The oscillations are sinusoidal as a function of time, and the period is  $T = 2\pi\sqrt{m/k}$ , independent of amplitude. When oscillations are small enough for these statements to be good approximations, we refer to them the oscillations as *simple harmonic*.

### **Chapter 3, Conservation of Momentum, page 129**

Since the kinetic energy of a material object depends on  $v^2$ , it isn't obvious that conservation of energy is consistent with Galilean relativity. Even if a certain mechanical system conserves energy in one frame of reference, the velocities involved will be different as measured in another frame, and therefore so will the kinetic energies. It turns out that consistency is achieved only if there is a new conservation law, conservation of *momentum*,

$$\mathbf{p} = m\mathbf{v}.$$

In one dimension, the direction of motion is described using positive and negative signs of the velocity  $\mathbf{v}$ , and since mass is always positive, the momentum carries the same sign. Thus conservation of momentum, unlike conservation of energy, makes direct predictions about the direction of motion. Although this line of argument was based on the assumption of a mechanical system, momentum need not be mechanical. Light has momentum.

A moving object's momentum equals the sum of the momenta of all its atoms. To avoid having to carry out this sum, we can use the concept of the *center of mass*. The center of mass can be defined as a kind of weighted average of the positions of all the atoms in the object,

$$\mathbf{x}_{cm} = \frac{\sum m_j \mathbf{x}_j}{\sum m_j},$$

and although the definition does involve a sum, we can often locate the center of mass by symmetry or by physically determining an object's balance point. The total momentum of the object is then given by

$$\mathbf{p}_{total} = m_{total} \mathbf{v}_{cm}.$$

The rate of transfer of momentum is called *force*,  $\mathbf{F} = d\mathbf{p}/dt$ , and is measured in units of newtons,  $1 \text{ N} = 1 \text{ kg}\cdot\text{m}/\text{s}^2$ . As a direct consequence of conservation of momentum, we have the following statements, known as *Newton's laws of motion*:

If the total force on an object is zero, it remains in the same state of motion.

$$\mathbf{F} = d\mathbf{p}/dt$$

Forces always come in pairs: if object A exerts a force on object B, then object B exerts a force on object A which is the same strength, but in the opposite direction.

Although the fundamental forces at the atomic level are gravity, electromagnetism, and nuclear forces, we use a different and more practical classification scheme in everyday situations. In this scheme, the forces between solid objects are described as follows:

<i>A normal force, <math>F_n</math>,</i>	is perpendicular to the surface of contact, and prevents objects from passing through each other by becoming as strong as necessary (up to the point where the objects break). "Normal" means perpendicular.
<i>Static friction, <math>F_s</math>,</i>	is parallel to the surface of contact, and prevents the surfaces from starting to slip by becoming as strong as necessary, up to a maximum value of $F_{s,max}$ . "Static" means not moving, i.e. not slipping.
<i>Kinetic friction, <math>F_k</math>,</i>	is parallel to the surface of contact, and tends to slow down any slippage once it starts. "Kinetic" means moving, i.e. slipping.

*Work* is defined as the transfer of energy by a force. ("By a force" is meant to exclude energy transfer by heat conduction.) The *work theorem* states that when a force occurs at a single point of contact, the amount of energy transferred by that force is given by  $dW = \mathbf{F} \cdot d\mathbf{x}$ , where  $d\mathbf{x}$  is the distance traveled by the point of contact. The *kinetic energy theorem* is  $dK_{cm} = \mathbf{F}_{total} \cdot d\mathbf{x}_{cm}$ , where  $dK_{cm}$  is the change in the energy,  $(1/2)mv_{cm}^2$ , an object possesses due to the motion of its center of mass,  $\mathbf{F}_{total}$  is the total force acting on the object, and  $d\mathbf{x}_{cm}$  is the distance traveled by the center of mass.

The relationship between force and interaction energy is  $U = -dF/dx$ . Any interaction can be described either by giving the force as a function of distance or the interaction energy as a function of distance; the other quantity can then be found by integration or differentiation.

An oscillator subject to friction will, if left to itself, suffer a gradual decrease in the amplitude of its motion as mechanical energy is transformed into heat. The *quality factor*,  $Q$ , is defined as

the number of oscillations required for the mechanical energy to fall off by a factor of  $e^{2\pi} \approx 535$ . To maintain an oscillation indefinitely, an external force must do work to replace this energy. We assume for mathematical simplicity that the external force varies sinusoidally with time,  $F = F_m \sin \omega t$ . If this force is applied for a long time, the motion approaches a steady state, in which the oscillator's motion is sinusoidal, matching the driving force in frequency but not in phase. The amplitude of this steady-state motion,  $A$ , exhibits the phenomenon of *resonance*: the amplitude is maximized at a driving frequency which, for large  $Q$ , is essentially the same as the natural frequency of the free vibrations,  $\omega_f$  (and for large  $Q$  this is also nearly the same as  $\omega_o = \sqrt{k/m}$ ). When the energy of the steady-state oscillations is graphed as a function of frequency, both the height and the width of the resonance peak depend on  $Q$ . The peak is taller for greater  $Q$ , and its full width at half-maximum is  $\Delta\omega \approx \omega_o/Q$ . For small values of  $Q$ , all these approximations become worse, and at  $Q < 1/2$  qualitatively different behavior sets in.

For three-dimensional motion, a moving object's motion can be described by three different velocities,  $v_x = dx/dt$ , and similarly for  $v_y$  and  $v_z$ . Thus conservation of momentum becomes three different conservation laws: conservation of  $p_x = mv_x$ , and so on. The principle of *rotational invariance* says that the laws of physics are the same regardless of how we change the orientation of our laboratory: there is no preferred direction in space. As a consequence of this, no matter how we choose our  $x$ ,  $y$ , and  $z$  coordinate axes, we will still have conservation of  $p_x$ ,  $p_y$ , and  $p_z$ . To simplify notation, we define a momentum *vector*,  $\mathbf{p}$ , which is a single symbol that stands for all the momentum information contained in the components  $p_x$ ,  $p_y$ , and  $p_z$ . The concept of a vector is more general than its application to the momentum: any quantity that has a direction in space is considered a vector, as opposed to a *scalar* like time or temperature. The following table summarizes some vector operations.

operation	definition
$ \mathbf{vector} $	$\sqrt{vector_x^2 + vector_y^2 + vector_z^2}$
$\mathbf{vector} + \mathbf{vector}$	Add component by component.
$\mathbf{vector} - \mathbf{vector}$	Subtract component by component.
$\mathbf{vector} \cdot \text{scalar}$	Multiply each component by the scalar.
$\mathbf{vector} / \text{scalar}$	Divide each component by the scalar.

Differentiation and integration of vectors is defined component by component.

There is only one meaningful (rotationally invariant) way of defining a multiplication of vectors whose result is a scalar, and it is known as the vector *dot product*:

$$\begin{aligned} \mathbf{b} \cdot \mathbf{c} &= b_x c_x + b_y c_y + b_z c_z \\ &= |\mathbf{b}| |\mathbf{c}| \cos \theta_{bc}. \end{aligned}$$

The dot product has most of the usual properties associated with multiplication, except that there is no "dot division."

## Chapter 4, Conservation of Angular Momentum, page 245

*Angular momentum* is a conserved quantity. For motion confined to a plane, the angular momentum of a material particle is

$$L = mv_{\perp} r,$$

where  $r$  is the particle's distance from the point chosen as the axis, and  $v_{\perp}$  is the component of its velocity vector that is perpendicular to the line connecting the particle to the axis. The choice of axis is arbitrary. In a plane, only two directions of rotation are possible, clockwise and counterclockwise. One of these is considered negative and the other positive. Geometrically,

angular momentum is related to rate at which area is swept out by the line segment connecting the particle to the axis.

*Torque* is the rate of change of angular momentum,  $\tau = dL/dt$ . The torque created by a given force can be calculated using any of the relations

$$\begin{aligned}\tau &= rF \sin \theta_{rF} \\ &= rF_{\perp} \\ &= r_{\perp}F,\end{aligned}$$

where the subscript  $\perp$  indicates a component perpendicular to the line connecting the axis to the point of application of the force.

In the special case of a *rigid body* rotating in a single plane, we define

$$\omega = \frac{d\theta}{dt} \quad [\text{angular velocity}]$$

and

$$\alpha = \frac{d\omega}{dt}, \quad [\text{angular acceleration}]$$

in terms of which we have

$$L = I\omega$$

and

$$\tau = I\alpha,$$

where the *moment of inertia*,  $I$ , is defined as

$$I = \sum m_i r_i^2,$$

summing over all the atoms in the object (or using calculus to perform a continuous sum, i.e. an integral). The relationship between the angular quantities and the linear ones is

$v_t = \omega r$	[tangential velocity of a point]
$v_r = 0$	[radial velocity of a point]
$a_t = \alpha r.$	[radial acceleration of a point]
	at a distance $r$ from the axis]
$a_r = \omega^2 r$	[radial acceleration of a point]
	at a distance $r$ from the axis]

In three dimensions, torque and angular momentum are vectors, and are expressed in terms of the vector *cross product*, which is the only rotationally invariant way of defining a multiplication of two vectors that produces a third vector:

$$\begin{aligned}\mathbf{L} &= \mathbf{r} \times \mathbf{p} \\ \boldsymbol{\tau} &= \mathbf{r} \times \mathbf{F}\end{aligned}$$

In general, the cross product of vectors  $\mathbf{b}$  and  $\mathbf{c}$  has magnitude

$$|\mathbf{b} \times \mathbf{c}| = |\mathbf{b}| |\mathbf{c}| \sin \theta_{bc},$$

which can be interpreted geometrically as the area of the parallelogram formed by the two vectors when they are placed tail-to-tail. The direction of the cross product lies along the line which is perpendicular to both vectors; of the two such directions, we choose the one that is right-handed, in the sense that if we point the fingers of the flattened right hand along  $\mathbf{b}$ , then bend the knuckles to point the fingers along  $\mathbf{c}$ , the thumb gives the direction of  $\mathbf{b} \times \mathbf{c}$ . In terms of components, the cross product is

$$\begin{aligned}(\mathbf{b} \times \mathbf{c})_x &= b_y c_z - c_y b_z \\(\mathbf{b} \times \mathbf{c})_y &= b_z c_x - c_z b_x \\(\mathbf{b} \times \mathbf{c})_z &= b_x c_y - c_x b_y\end{aligned}$$

The cross product has the disconcerting properties

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a} \quad [\text{noncommutative}]$$

and

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \neq (\mathbf{a} \times \mathbf{b}) \times \mathbf{c} \quad [\text{nonassociative}],$$

and there is no “cross-division.”

For rigid-body rotation in three dimensions, we define an angular velocity vector  $\boldsymbol{\omega}$ , which lies along the axis of rotation and bears a right-hand relationship to it. Except in special cases, there is no scalar moment of inertia for which  $\mathbf{L} = I\boldsymbol{\omega}$ ; the moment of inertia must be expressed as a matrix.

## Chapter 5, Thermodynamics, page 299

A fluid is any gas or liquid, but not a solid; fluids do not exhibit shear forces. A fluid in equilibrium exerts a force on any surface which is proportional to the surface’s area and perpendicular to the surface. We can therefore define a quantity called the *pressure*,  $P$ , which is ratio of force to area,

$$P = \frac{F_{\perp}}{A},$$

where the subscript  $\perp$  indicates the component of the fluid’s force which is perpendicular to the surface.

Usually it is only the difference in pressure between the two sides of a surface that is physically significant. Pressure doesn’t just “press down” on things; air pressure upward under your chin is the same as air pressure downward on your shoulders. In a fluid acted on by gravity, pressure varies with depth according to the equation

$$dP = \rho \mathbf{g} \cdot d\mathbf{y}.$$

This equation is only valid if the fluid is in equilibrium, and if  $g$  and  $r$  are constant with respect to height.

Temperature can be defined according to the volume of an ideal gas under conditions of standard pressure. The Kelvin scale of temperature used throughout this book equals zero at

absolute zero, the temperature at which all random molecular motion ceases, and equals 273 K at the freezing point of water. We can get away with using the Celsius scale as long as we are only interested in temperature differences; a difference of 1 degree C is the same as a difference of 1 degree K.

It is an observed fact that ideal gases obey the *ideal gas law*,

$$PV = nkT,$$

and this equation can be explained by the kinetic theory of heat, which states that the gas exerts pressure on its container because its molecules are constantly in motion. In the kinetic theory of heat, the temperature of a gas is proportional to the average energy per molecule.

Not all the heat energy in an object can be extracted to do mechanical work. We therefore describe heat as a lower grade of energy than other forms of energy. Entropy is a measure of how much of a system's energy is inaccessible to being extracted, even by the most efficient heat engine; a high entropy corresponds to a low grade of energy. The change in a system's entropy when heat  $Q$  is deposited into it is

$$\Delta S = \frac{Q}{T}.$$

The efficiency of any heat engine is defined as

$$\text{efficiency} = \frac{\text{energy we get in useful form}}{\text{energy we pay for}},$$

and the efficiency of a Carnot engine, the most efficient of all, is

$$\text{efficiency} = 1 - \frac{T_L}{T_H}.$$

These results are all closely related. For instance, example 11 on page 315 uses  $\Delta S = Q/T$  and  $\text{efficiency} = 1 - T_L/T_H$  to show that a Carnot engine doesn't change the entropy of the universe.

Fundamentally, entropy is defined as the being proportional to the natural logarithm of the number of states available to a system, and the above equation then serves as a definition of temperature. The entropy of a closed system always increases; this is the second law of thermodynamics.

## Chapter 6, Waves, page 343

*Wave motion* differs in three important ways from the motion of material objects:

Waves obey the principle of superposition. When two waves collide, they simply add together.

The medium is not transported along with the wave. The motion of any given point in the medium is a vibration about its equilibrium location, not a steady forward motion.

The velocity of a wave depends on the medium, not on the amount of energy in the wave. (For some types of waves, notably water waves, the velocity may also depend on the shape of the wave.)

Sound waves consist of increases and decreases (typically very small ones) in the density of the air. Light is a wave, but it is a vibration of electric and magnetic fields, not of any physical medium. Light can travel through a vacuum.

A periodic wave is one that creates a periodic motion in a receiver as it passes it. Such a wave has a well-defined period and frequency, and it will also have a wavelength, which is the distance in space between repetitions of the wave pattern. The velocity, frequency, and wavelength of a periodic wave are related by the equation

$$v = f\lambda.$$

A wave emitted by a moving source will undergo a *Doppler shift* in wavelength and frequency. The shifted wavelength is given by the equation

$$\lambda' = \left(1 - \frac{u}{v}\right) \lambda,$$

where  $v$  is the velocity of the waves and  $u$  is the velocity of the source, taken to be positive or negative so as to produce a Doppler-lengthened wavelength if the source is receding and a Doppler-shortened one if it approaches. A similar shift occurs if the observer is moving, and in general the Doppler shift depends approximately only on the relative motion of the source and observer if their velocities are both small compared to the waves' velocity. (This is not just approximately but exactly true for light waves, as required by Einstein's theory of relativity.)

Whenever a wave encounters the boundary between two media in which its speeds are different, part of the wave is reflected and part is transmitted. The reflection is always reversed front-to-back, but may also be inverted in amplitude. Whether the reflection is inverted depends on how the wave speeds in the two media compare, e.g. a wave on a string is uninverted when it is reflected back into a segment of string where its speed is lower. The greater the difference in wave speed between the two media, the greater the fraction of the wave energy that is reflected. Surprisingly, a wave in a dense material like wood will be strongly reflected back into the wood at a wood-air boundary.

A one-dimensional wave confined by highly reflective boundaries on two sides will display motion which is periodic. For example, if both reflections are inverting, the wave will have a period equal to twice the time required to traverse the region, or to that time divided by an integer. An important special case is a sinusoidal wave; in this case, the wave forms a stationary pattern composed of a superposition of sine waves moving in opposite direction.

## Chapter 7, Relativity, page 385

Experiments show that space and time do not have the properties claimed by Galileo and Newton. Time and space as seen by one observer are distorted compared to another observer's perceptions if they are moving relative to each other. This distortion is quantified by the factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where  $v$  is the relative velocity of the two observers, and  $c$  is a universal velocity that is the same in all frames of reference. Light travels at  $c$ . A clock appears to run fastest to an observer who is not in motion relative to it, and appears to run too slowly by a factor of  $\gamma$  to an observer who has a velocity  $v$  relative to the clock. Similarly, a meter-stick appears longest to an observer who sees it at rest, and appears shorter to other observers. Time and space are relative, not absolute. In particular, there is no well-defined concept of simultaneity.

All of these strange effects, however, are very small when the relative velocities are small compared to  $c$ . This makes sense, because Newton's laws have already been thoroughly tested by experiments at such speeds, so a new theory like relativity must agree with the old one in their realm of common applicability. This requirement of backwards-compatibility is known as the correspondence principle.

Relativity has implications not just for time and space but also for the objects that inhabit time and space. The correct relativistic equation for momentum is

$$p = m\gamma v,$$

which is similar to the classical  $p = mv$  at low velocities, where  $\gamma \approx 1$ , but diverges from it more and more at velocities that approach  $c$ . Since  $\gamma$  becomes infinite at  $v = c$ , an infinite force would be required in order to give a material object enough momentum to move at the speed of light. In other words, material objects can only move at speeds lower than  $c$ . Relativistically, mass and energy are not separately conserved. Mass and energy are two aspects of the same phenomenon, known as mass-energy, and they can be converted to one another according to the equation

$$E = mc^2.$$

The mass-energy of a moving object is  $\mathcal{E} = m\gamma c^2$ . When an object is at rest,  $\gamma = 1$ , and the mass-energy is simply the energy-equivalent of its mass,  $mc^2$ . When an object is in motion, the excess mass-energy, in addition to the  $mc^2$ , can be interpreted as its kinetic energy.

## Chapter 8, Atoms and Electromagnetism, page 459

All the forces we encounter in everyday life boil down to two basic types: gravitational forces and electrical forces. A force such as friction or a "sticky force" arises from electrical forces between individual atoms.

Just as we use the word mass to describe how strongly an object participates in gravitational forces, we use the word *charge* for the intensity of its electrical forces. There are two types of charge. Two charges of the same type repel each other, but objects whose charges are different attract each other. Charge is measured in units of coulombs (C).

*Mobile charged particle model:* A great many phenomena are easily understood if we imagine matter as containing two types of charged particles, which are at least partially able to move around.

*Positive and negative charge:* Ordinary objects that have not been specially prepared have both types of charge spread evenly throughout them in equal amounts. The object will then tend not to exert electrical forces on any other object, since any attraction due to one type of charge will be balanced by an equal repulsion from the other. (We say "tend not to" because bringing the object near an object with unbalanced amounts of charge could cause its charges to separate from each other, and the force would no longer cancel due to the unequal distances.) It therefore makes sense to describe the two types of charge using positive and negative signs, so that an unprepared object will have zero *total* charge.

The *Coulomb force law* states that the magnitude of the electrical force between two charged particles is given by

$$|F| = \frac{k|q_1||q_2|}{r^2}.$$

*Conservation of charge:* An even more fundamental reason for using positive and negative



signs for charge is that with this definition the total charge of a closed system is a conserved quantity.

*Quantization of charge:* Millikan's oil drop experiment showed that the total charge of an object could only be an integer multiple of a basic unit of charge,  $e$ . This supported the idea that the "flow" of electrical charge was the motion of tiny particles rather than the motion of some sort of mysterious electrical fluid.

Einstein's analysis of Brownian motion was the first definitive proof of the existence of atoms. Thomson's experiments with vacuum tubes demonstrated the existence of a new type of microscopic particle with a very small ratio of mass to charge. Thomson correctly interpreted these as building blocks of matter even smaller than atoms: the first discovery of subatomic particles. These particles are called electrons.

The above experimental evidence led to the first useful model of the interior structure of atoms, called the raisin cookie model. In the raisin cookie model, an atom consists of a relatively large, massive, positively charged sphere with a certain number of negatively charged electrons embedded in it.

Rutherford and Marsden observed that some alpha particles from a beam striking a thin gold foil came back at angles up to 180 degrees. This could not be explained in the then-favored raisin-cookie model of the atom, and led to the adoption of the planetary model of the atom, in which the electrons orbit a tiny, positively-charged nucleus. Further experiments showed that the nucleus itself was a cluster of positively-charged protons and uncharged neutrons.

Radioactive nuclei are those that can release energy. The most common types of radioactivity are alpha decay (the emission of a helium nucleus), beta decay (the transformation of a neutron into a proton or vice-versa), and gamma decay (the emission of a type of very-high-frequency light). Stars are powered by nuclear fusion reactions, in which two light nuclei collide and form a bigger nucleus, with the release of energy.

Human exposure to ionizing radiation is measured in units of millirem. The typical person is exposed to about 100 mrem worth of natural background radiation per year.

## Chapter 9, DC Circuits, page 515

All electrical phenomena are alike in that they arise from the presence or motion of charge. Most practical electrical devices are based on the motion of charge around a complete circuit, so that the charge can be recycled and does not hit any dead ends. The most useful measure of the flow of charge is *current*,

$$I = \frac{dq}{dt}.$$

An electrical device whose job is to transform energy from one form into another, e.g. a lightbulb, uses power at a rate which depends both on how rapidly charge is flowing through it and on how much work is done on each unit of charge. The latter quantity is known as the voltage difference between the point where the current enters the device and the point where the current leaves it. Since there is a type of electrical energy associated with electrical forces, the amount of work they do is equal to the difference in potential energy between the two points, and we therefore define voltage differences directly in terms of electrical energy,

$$\Delta V = \frac{\Delta U_{elec}}{q}.$$

The rate of power dissipation is

$$P = I\Delta V.$$

Many important electrical phenomena can only be explained if we understand the mechanisms of current flow at the atomic level. In metals, currents are carried by electrons, in liquids by ions. Gases are normally poor conductors unless their atoms are subjected to such intense electrical forces that the atoms become ionized.

Many substances, including all solids, respond to electrical forces in such a way that the flow of current between two points is proportional to the voltage difference between those points (assuming the voltage difference is small). Such a substance is called ohmic, and an object made out of an ohmic substance can be rated in terms of its resistance,

$$R = \frac{\Delta V}{I}$$

An important corollary is that a perfect conductor, with  $R = 0$ , must have constant voltage everywhere within it.

A schematic is a drawing of a circuit that standardizes and stylizes its features to make it easier to understand. Any circuit can be broken down into smaller parts. For instance, one big circuit may be understood as two small circuits in series, another as three circuits in parallel. When circuit elements are combined in parallel and in series, we have two basic rules to guide us in understanding how the parts function as a whole:

*The junction rule:* In any circuit that is not storing or releasing charge, conservation of charge implies that the total current flowing out of any junction must be the same as the total flowing in.

*The loop rule:* Assuming the standard convention for plus and minus signs, the sum of the voltage drops around any closed loop in a circuit must be zero.

The simplest application of these rules is to pairs of resistors combined in series or parallel. In such cases, the pair of resistors acts just like a single unit with a certain resistance value, called their equivalent resistance. Resistances in series add to produce a larger equivalent resistance,

$$R = R_1 + R_2,$$

because the current has to fight its way through both resistances. Parallel resistors combine to produce an equivalent resistance that is smaller than either individual resistance,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2},$$

because the current has two different paths open to it.

An important example of resistances in parallel and series is the use of voltmeters and ammeters in resistive circuits. A voltmeter acts as a large resistance in parallel with the resistor across which the voltage drop is being measured. The fact that its resistance is not infinite means that it alters the circuit it is being used to investigate, producing a lower equivalent resistance. An ammeter acts as a small resistance in series with the circuit through which the current is to be determined. Its resistance is not quite zero, which leads to an increase in the resistance of the circuit being tested.

## **Chapter 10, Fields, page 563**

Newton conceived of a universe where forces reached across space instantaneously, but we now