

Spotter

Benjamin Crowell

www.lightandmatter.com

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1 Introduction

purpose Spotter is computer software for checking students' answers to math and science problems. For the student, the benefit of the system is that it can not only tell whether the answer is correct, but it can also help to diagnose an incorrect answer. Spotter isn't limited to numerical problems. For instance, if the problem is to solve the equation $x - b - 7a = 0$ for x , the student can type in either $b + 7a$ or $7a + b$ as the answer, and the program will know it's correct.

legalities Spotter is free software, and it comes with source code. It is copyright 2001 by Benjamin Crowell, and to have permission to copy it, you must agree to the terms of the licensing agreement on page 22. This documentation is copyright 2001 by Benjamin Crowell, and is available under the GFDL 1.1 license on page 29.

2 Instructions For Students

Spotter is set up as an interactive web page that you can access through any computer that has an internet connection and a web browser. Your instructor will tell you the web address to use. You don't need to install any software on your computer. Everything you need to know about using the software is in this section of the documentation; the later sections are for instructors.

The main thing you have to be careful about is the notation you use for inputting mathematical expressions. Spotter is designed to allow you to use something resembling normal human mathematical notation, as opposed to the notation used in computer programs. However, human math notation is designed for humans, not computers, and you need to learn a few things about how to type your expressions in a form that

Spotter will interpret correctly.

First, everything you type will be smashed down to one line of text, eliminating the superscripts and subscripts. For example, a variable name with a subscript, like x_1 , is entered as `x1`. Since there are no superscripts, you have to enter exponents using the `^` symbol (shift-6), e.g. x^2 becomes `x^2`. You can enter a square root as either `sqrt(x)` or `x^.5`. There is no way to enter the times symbol, \times , without confusing the computer and making it think you meant the variable x , so in scientific notation you should simply leave a space where you would normally put the times symbol, e.g. 5×10^6 becomes `5 10^6`. Don't try to enter this as `5e+6`; that's what a lot of computer software would want, but Spotter is trying to interpret everything as normal human notation, so it will think you meant $5e + 6$, where e is a variable.

Another thing to keep in mind is that human languages, including human math notation, are ambiguous. Use parentheses liberally to make your meaning clear. There are two main situations where you need to watch out. First, arguments to functions: `sin 2x` will be interpreted as $(\sin 2)(x)$; if you intended $\sin(2x)$, you should have entered `sin(2x)`. Second, the bottom of fractions: `1/3c` will be interpreted as $(1/3)c$, so if you want $\frac{1}{3c}$, you need to enter `1/(3c)`.

Finally, you need to know a tiny bit about how Spotter works, or you may get a nasty surprise in certain situations. Spotter works by comparing preprogrammed answers with yours, and the comparison is done numerically, not symbolically. For instance, if your instructor put in the answer $b + 7a$ and you put in $7a + b$, the software will randomly pick values for the variables a and b , compute both results, and see if they came out the same. If it does this a few times, and the answers always match, then it assumes they're equivalent mathematical expressions. So far so good. The pitfall comes when you're assigned a problem where you're supposed to put the answer in a particular form. For instance, you may be assigned to simplify the expression $3x + 5x + 7$. Your instructor puts in the answer, $8x + 7$. Now if you put in $8x + 7$ or $7 + 8x$, Spotter figures out that you're right, and tells you so. But if you put in $6x + 2x + 7$, it will also tell you you're right, since this is numerically the same as $8x + 7$. It tells you you're right, but you're wrong, because your form isn't any simpler than the original form, and your job was to simplify. It's your responsibility to realize this — don't try to blame it on the software! In general, it's up to you to check whether your answer has the right *form*; Spotter only checks whether it's *numerically* correct.

3 Setup

If you're a student using Spotter, you don't need to read this! These instructions are for instructors who are setting the software up for their students. In what follows, I assume your server is a Unix machine, and that you are somewhat familiar with the Unix command line. If you're using a Windows server, I can't help you, but if you're knowledgeable enough, you can probably find the Windows equivalents of these steps.

a test setup I recommend that you start by installing the software on a machine on your own desktop. That way you can test the software, write your own input file, and make sure everything works before trying to get it going on

your school or webhost's server. On a Mac, do `sudo apachectl start`. On a Debian Linux system, do the command `apt-get install apache`.

- perl** Make sure Perl 5.6 or later is installed on your system.
- download Spotter** Download the source code. Try running the calculator program that comes with Spotter, using the command `./Calc.pl`. If it doesn't run, you probably don't have Perl installed correctly, or you have an older version of Perl, or your installation is missing some of the standard libraries. If you want to, you can give the mathematical routines a workout by running a test suite through the calculator, using the following command:
- ```
make test
```
- The comments at the top of the `tests/testsuite` input file will tell you how to interpret the output.
- privileges** If you're installing on your own Unix machine, do `sudo tcsh` on MacOS X, or `su` on Linux, so that you have privileges.
- installing the CGI** There is some stuff near the top of the file `Makefile` that is set appropriately for the Apache web server running on Debian or Ubuntu linux, but may need to be changed for other web servers or operating systems.
- ```
CGI_GENERAL = /usr/lib/cgi-bin
```
- should be the cgi-bin on your machine (e.g., `/Library/WebServer/CGI-Executables` on MacOS).
- ```
WEB_SERVER_GROUP = www-data
```
- should be the group that your web server runs in (`_www` on MacOS).
- ```
WEB_SERVER_DATA = /var/www/html
```
- should be the directory where your web server stores the documents it serves (`/Library/WebServer/Documents`).
- The file `config.json` contains a variety of configuration parameters. The two that are most likely to be of interest are `language` (set to English, `en`, by default) and `immune_ip_range`, which specifies a block of IP addresses for your own school, which will never be blocked from accessing Spotter even if an excessive number of requests is received within a short time. For example, the unix command `dig +short fullcoll.edu` tells me that my school's web site is at `207.233.83.8`, so I set `immune_ip_range` to `207.233`, which should cover any machines on my campus.
- Do the command `make install`.
- The sample answer file `sample.xml` will have been automatically placed in `/usr/lib/cgi-bin/spotter3/answers`. Your own real answer files should go in the same place.
- enable CGI** You may need to do something special to enable CGI. For example, with `apache2` on linux (and possibly on MacOS?) you probably need to do this:
- ```
a2enmod cgi
```
- ```
service apache2 restart
```
- On MacOS X, edit the file `/etc/apache2/httpd.conf` and uncomment the line
- ```
#AddHandler cgi-script .cgi
```
- In the same file, add the following:
- ```
1 <Directory "/Library/WebServer/CGI-Executables">
2     Options ExecCGI
```

```
3     AddHandler cgi-script .cgi
4 </Directory>
```

libraries You need to install the following perl libraries:

```
XML::Parser XML::Simple JSON CGI::Application CGI::Session
CGI::Application::Plugin::Authentication Data::Dumper Carp::Always
```

On Debian or Ubuntu Linux, you can do this with the command `make depend`. On a system such as debian stable with older packages, you may find that one or more of these is unavailable. In that case, do something like this:

```
cpan CGI::Application::Plugin::Authentication
```

On other systems, use that system's software packaging system, or use `cpan` as in the example above.

the answer file and the log file

If your answer file is called `spotter.xml`, then by default your log file will be called `spotter.log`. It will be created in the subdirectory `spotter3/data/log`, inside the `cgi-bin` directory. If you make mistakes in your answer file that cause errors at runtime, the error messages will show up in this file. (They are not displayed in the browser. This is a security feature meant to keep the software from inadvertently divulging information about the answer file.)

running the CGI

If you're running Spotter on your desktop machine, the URL you use will be something like this:

```
http://localhost/cgi-bin/spotter3/Spotter.cgi?what=check&file=sample
```

If you're running it on a machine elsewhere, replace `localhost` with the relevant domain name. You can also try changing `?what=check` to `?what=check&debug=`, which will make the CGI print out a little more information, such as the name of the log file.

The server gives a generic error message when Spotter.cgi runs, saying that there's something wrong with the software, and you should contact the webmaster. Try running `Spotter.cgi` from the command line. When it asks for pairs of arguments, type control-D and hit return. If the compiler gives an error, it may be because you are running an old version of Perl, or don't have some of the usual modules installed. Another possibility is that you forgot to change the `use lib` statement, and it refers to a directory on your development machine rather than on the server.

Spotter.cgi prints the title, and nothing more. The program has crashed. This could happen if it can't find `spotter.xml` in the `cgi-bin` directory.

Spotter.cgi used to work, but now crashes. Something you've done to your answer file has caused it to get upset. See the section on troubleshooting answer files on page 13.

If none of the above suggestions get you going, try to find out where your server's error log is, and look at it for clues. On MacOS X, it's in `/var/log/httpd/error_log`. On Debian with Apache 2 installed, it's `/var/log/apache2/error.log`.

As a last resort, if you can't get access to the error logs, you can uncomment the line

```
#my $my_query_string = "file=lm&what=check";
```

in the subroutine `decode_pars` in the `Url` class in `Spotter.cgi`. This

allows you to run Spotter from the command line and provide it with the parameters that were causing it to crash. (You're supposed to be able to do this by typing in the parameters when it prompts you, but I've never been able to get that to work.) You'll be able to see any error messages it prints out.

4 Math Notation

The following is a full specification of Spotter's notation, but most people learn best by example, so you may want to look at the file `testsuite`, which is meant to be run in the calculator program, and try various examples yourself in the calculator.

characters Variable names can include roman letters, greek letters, digits, underscores, and primes (represented with the apostrophe, `'`, not the backtick, ```). The first character in a variable name must be a letter. Most users have no way to generate greek letters at the keyboard, so when setting up an answer file, it's best to spell them out, e.g. `tau` instead of `τ`. In addition to the characters that are legal parts of variable names, the following are also legal characters in input: `+ - * / ^ ! () [] { } | , . < > ; = ?` and whitespace characters (spaces, tabs, and newlines). Spotter is case-sensitive.

operators The following is a list of the operators, in order of precedence:

functions	Functions have the highest precedence.
<code>^</code> or <code>**</code>	exponentiation
<code>*</code> / <code>mod</code>	multiplication, division, mod
<code>+</code> -	binary and unary addition and subtraction
<code>-></code>	conversion, e.g. <code>(1 m)->cm</code> gives 100 cm
<code>,</code>	separates arguments to functions of two variables
<code>eq ne</code>	equals, doesn't equal
<code>not</code>	logical negation
<code>and</code>	logical and
<code>or xor</code>	logical or, exclusive or

All the operators are left-associative except for exponentiation, just as in human math notation. The multiplication operator `*` is always optional. It can be replaced by a space or omitted entirely, as in human math notation. Sometimes it makes a difference whether you use a space or not. For instance, if you have a variable named `m`, then `2m` means $(2)(m)$, while `2 m` means two meters. It's an error to write an expression like `x2` using implied multiplication of a variable on the left and a number on the right; this is to protect against cases where someone tries to write x^2 as `x2`.

Multiplication of a number by a unit, as in `2 m`, has a higher precedence than other kinds of multiplication and division, so `1 m/1 s` is interpreted as one meter per second, not as $((1 m)/1)s$. However, it's not a good idea to depend on this feature. You're much more likely to get the results you intended if you surround number-unit expressions with parentheses. For instance, `1/12 ft` produces `0.083333333333333333 ft-1`, with units of inverse feet, rather than the intended result of one inch.

The logical operators require unitless operands. Zero means false, and any other number means true. The equality operator `a eq b` tests for $|a - b| < \epsilon \max(|a|, |b|)$, where ϵ is a measure of the machine's floating-point precision, computed at runtime. When used with units, the `eq` operator

tells whether they are identical, e.g. `kg eq kg` is true, but `kg eq g` is false. To test whether two units measure the same kind of thing, use the `base_units` function, e.g. `base_units(1 mm) eq base_units(1 ft)` is true.

units Numbers with units can be written in the natural way, e.g. `37 cm`. To avoid ambiguities, it is best to surround all such expressions with parentheses. Units can stand on their own, and it is possible to do computations with bare units, e.g. `(ft * ft)/in` gives a result of 3.6576 meters. Complicated units can be indicated with expressions such as `m2` (square meters), `N.m`, `N-m`, or `N*m` (newton-meters), `lb/in2` (pounds per square inch), or `m-3/2` (meters to the $-3/2$ power, as in the units of a wavefunction in quantum mechanics). A dot, dash, or asterisk is used for multiplication; there is no implied multiplication of units. In such expressions, the division operator has lower precedence than multiplication, so e.g. `J/N.m` is a unitless quantity.

parentheses The three forms of parentheses, `(...)`, `[...]`, and `{...}`, can all be used interchangeably and nested inside one another. The absolute value signs `|...|` can be used in the same way as the parentheses; when used around a complex number, they indicate the magnitude of the number. Parentheses surrounding the arguments of functions are optional; the following are all legal: `sinx`, `sin x`, `sin(x)`, `sin[x]`, `sin{x}`, and `sin|x|`. Parentheses should be used when the argument is an expression, e.g. `sin 2x` will be interpreted as `(sin 2)(x)`, not as `sin(2x)`.

built-in functions, constants, and units The following is a description of all the available built-ins. The instructor can disable some of these for particular problems. For instance, if a problem involves a variable `e`, one may want to disable the built-in constant `e` (base of natural logarithms).

Constants:

```
pi  pi = 3.1...
e   e = 2.7...
i   sqrt(-1)
```

Units:

```
m      meters    g      grams
s,sec  seconds    C      coulombs
deg     degrees   N      newtons
J       joules    W      watts
Pa      pascals   Hz     hertz
V       volts    A      amperes
H       henries  F      farads
Ω,ohm  ohms       T      teslas
in      inches   mi     miles
min     minutes  hr     hours
ft      feet
```

Metric prefixes:

f 10⁻¹⁵
 p 10⁻¹²
 n 10⁻⁹
 μ,u 10⁻⁶
 m 10⁻³
 c 10⁻²
 k 10³
 M 10⁶
 G 10⁹

Functions

<code>exp</code>	e^x
<code>ln</code>	natural logarithm
<code>log</code>	base-10 logarithm
<code>log10</code>	base-10 logarithm
<code>sqrt</code>	square root
<code>sin, cos, tan, sec, csc, cot</code>	trig functions, with arguments in radians
<code>asin, acos, atan</code>	inverse trig functions
<code>sinh, cosh, tanh, sech, csch, coth</code>	hyperbolic trig functions
<code>asinh, acosh, atanh</code>	inverse hyperbolic trig functions
<code>abs, Re, Im, arg, conj</code>	magnitude, real part, imaginary part, argument, and conjugate
<code>!, !!, Gamma, ln_Gamma</code>	factorial, odd factorial, Γ function, natural log of the Γ function
<code>atomize</code>	converts a quantity to metric meter-gram-second-coulomb form
<code>units</code>	strips off the number and leaves the units
<code>base_units</code>	like <code>units(atomize(...))</code>

errors All the functions accept complex arguments and give complex results when necessary, so e.g. `ln -1` is not an error. An error does occur when the function blows up to infinity, or in expressions like `0/0` or `00`. An undefined result is output as the symbol `?`.

Most functions require a unitless argument and produce a unitless result. (If the argument can be converted to unitless form, it will be, and no error will result, e.g. `asin(1 in/1 ft)` results in $\sin^{-1}(1/12) = 0.083\dots$) The logarithm and `arg` functions accept arguments that have units, and strip them of their units, producing a unitless result. The `Re`, `Im`, `conj`, and `abs` functions preserve the units of their arguments. The functions `sqrt`, `atomize`, `units`, and `base_units` manipulate the units of their arguments, and produce a result that has the expected units.

An exponentiation a^b requires that b be unitless. If a can be reduced to a unitless number, it is, e.g. `(200 cm/1 m)^3` produces 8 as a result. If a can't be reduced to a unitless number then b must be a rational number.

the calculator In the calculator program, an equals sign can be used to assign a value to a variable. A semicolon can be used to separate multiple calculations or assignments on the same line. The tilde, `~`, can be used to represent the result of the previous calculation. Use control-D to exit from the calculator. The calculator has command-line options, which are documented at the

top of the source code. A line can end with a comment, marked by a pound sign, #.

special values The symbols `undef` and `inf` stand can be used to create four special values. The following table shows the syntax for expressing those values, their mathematical meanings, and examples of problems for which they would be the correct answer.

<code>undef</code>	undefined	$\lim_{x \rightarrow \infty} \sin x$
<code>inf</code>	∞	$\lim_{x \rightarrow 0} 1/x$
<code>+inf</code>	$+\infty$	$\lim_{x \rightarrow \infty} \ln x$
<code>-inf</code>	$-\infty$	$\lim_{x \rightarrow 0^+} \ln x$

The four categories are modeled on the ones used by the computer algebra system Maxima. Values lower in the table can be thought of as more specific cases of the ones higher up. For example, `-inf` is also infinite and undefined. These four values can only be constructed using the very special syntaxes listed here (or some nearly identical ones such as `(-inf)`). For example, the expression `1/0` simply produces an error, not a result of `inf` or `undef`. Note that unary `+` is a nontrivial operation when applied to `inf`.

When the correct answer to a question is one of these special values, the student must match it exactly. For example, if the correct answer is `-inf`, a student answer of `inf` is incorrect. However, you can provide helpful feedback for that particular incorrect answer.

Filters cannot be used with special values. Special values can't have units.

5 Creating an Answer File

writing up a problem The following is a sample of how to write an entry in the answer file for a particular problem:

```
1 <problem id="deriv_cosine">
2   <find id="1">
3     The derivative of 3 cos(omega t) with respect to t.
4     <var sym="omega" units="s-1">the frequency</var>
5     <var sym="t" units="s">time</var>
6     <ans e="-3 omega sin(omega t)"/>
7     <ans e="-3 sin(omega t)">You forgot to apply the chain rule.</ans>
8   </find>
9 </problem>
```

If you know HTML, this will look familiar. The answer file is in a format called XML, which is closely related to HTML. (It's possible to write HTML that is valid XML.) Note how every tag `<blah ...>` has a matching end tag, `</blah>`. The only exception is that if there isn't anything inside the tag, as on line 6, then `<blah ...><blah ...>` can be abbreviated as `<blah .../>`.

Line 1 gives the problem a name, `deriv_cosine`. If this problem is number 7 in the book, then you would also need the following line near the top of the file:

```
<num id="deriv_cosine" label="7"/>
```


The `<num>` tag associates the symbolic name `deriv_cosine` with the number 7. Keeping all the problem numbers in one place makes it easier to renumber problems when necessary.

Line 2 says we're going to give information about one of the things the student is supposed to find in this problem. If a single problem has more than one quantity that the student is supposed to find, then we'd have `find` tags nested inside `problem` tags like this:

```
<problem ...><find...>...</find><find...>...</find></problem>
```

This problem only has one thing to find. The `id` attribute is simply an integer that is different for each set of `<find>` tags.

The text on line 3 is a description of what the student is supposed to find. For practical reasons, and perhaps for copyright reasons as well, it won't normally be a full statement of the problem; we assume the student has the book open on the table next to the computer. If you need to use superscripts in this description,¹ you can indicate them like this: `x{2}` produces x^2 . Greek letters can be specified like this: `e{delta}` gives δ , and `e{Delta}` gives Δ . (Other symbols that are defined as html entities can also be written in this way, e.g., `e{infin}` gives an infinity symbol.) You can do `i{italics}` and `b{boldface}`. You can include figures like this: `f{http://myserver.com/myimage.gif}`.

Lines 4 and 5 describe the variables the student is supposed to use in forming the answer. In many cases, the student will realize just by looking at this list that her answer is incorrect, because it includes variables it isn't supposed to include. If the variable is unitless, omit the `units` attribute. If the answer is going to involve one of the built-in constants (`pi`, `e`, or `i`), you may want to alert the student to this; otherwise she may think she is only allowed to type in symbols from the list of variables. You might think that for a purely symbolic problem, you might as well omit the units on all the variables, making them unitless. That would be a bad idea. Putting in the units has the advantage that students will get instant feedback when they input an answer that is nonsense in terms of units. Furthermore, you'll find that once in a while you'll make mistakes typing in the answers, and Spotter will catch them for you based on units.

Line 6 gives the correct answer. By leaving the inside of the tag completely empty, you're telling Spotter it's correct.

Line 7 gives an incorrect answer that we know from experience occurs frequently. The text inside will be displayed to any student who volunteers this answer.

Normally we only need to give one version of the correct answer. It isn't necessary to give multiple forms if they're numerically equal. It is possible to give more than one right answer, but in most cases that actually come up this can be handled simply by using the `filter` attribute described below.

Listing incorrect answers and hints to go with them is optional. Spotter will tell the student she is incorrect if she gives an answer that isn't numerically equal to any of the ones you thought to put in. Educationally, it's more effective to give a helpful hint for any wrong answers you anticipate

¹You can also do subscripts using an underbar in place of the caret, but I suggest you refrain. It's better to show the variable the same way you expect the student to input it.

will occur frequently.

There are some cases where Spotter will give a helpful hint even if you don't program one in. For instance, if the student enters an expression like $x + t$, where x has units of meters and t has units of seconds, Spotter will tell her that her units don't make sense.

a complete answer file The file `sample.xml` is a complete working answer file. The following is a listing of `sample.xml`.

```
1 <?xml version="1.0"?>
2 <spotter>
3
4 <num id="ohm" label="1"/>
5
6 <toc_level level="0" type="chapter"/>
7
8 <!-- ===== chapter 1 ===== -->
9 <toc type="chapter" num="1" title="Ohm's Law">
10 <!-- ===== Ohm's law -->
11 <problem id="ohm">
12 <find id="1">
13 <math>V=IR</math> for I.
14 <var sym="V" units="V">the voltage drop</var>
15 <var sym="R" units="ohm">the resistance</var>
16 <ans e="V/R"/>
17 </find>
18 </problem>
19 </toc>
20 </spotter>
```

Lines 8 and 10 are comments. Make sure not to use double dash anywhere inside such a comment, because a double dash is part of the markers for the beginning and end of the comment.

Lines 2 and 20 are mandatory, and must be the outermost tags in the whole file.

Line 6 defines a hierarchical organization with only one level. You can have more. For instance, if you want to have a single answer file for several books, then the outermost level, level 0, would be `book`, and the next level, 1, would be `chapter`.

All the other parts of the file have been discussed previously.

(The `log_file` tag used in versions $j=2.3.1$ is deprecated, and will be ignored if present.)

robustness How robust and reliable is Spotter's method for testing whether two expressions are equivalent? Will it ever say expressions are the same when they're different, or different when they're the same? How can we make sure the program won't fail because it chooses a random value for a variable that lies outside the domain of a certain function?

By default, Spotter will supply the variables with random real values which are uniformly distributed in the range from 0 to 1. Although this can be changed by putting `min="..."` and `max="..."` attributes inside the `<var>` tag, there is normally no need to fiddle around with this, even if the values

are physically unreasonable. For instance, it is not an error if the result of the calculation ends up being a complex number. Typically the answer is going to be an analytic function in the complex plane, perhaps with a few singularities, and there is zero probability that a randomly chosen set of variables will happen to lie right at one of these badly behaved points. Any analytic function is completely defined by specifying its behavior throughout a particular neighborhood, and in practice, a few randomly chosen points in a neighborhood are a sufficient test. All of this applies even if the problem has nothing to do with complex numbers; the complex number stuff is just an internal trick, hidden from the student, for testing whether two expressions are equivalent, without having to worry about the domains of functions. With no exceptions, all of Spotter's built-in functions have as their domain the entire complex plane (except for isolated singularities). It's perfectly legal, for instance, to calculate $0.5!$ or $\text{acos}(2)$.

What if your problem *does* explicitly involve complex numbers? If the expressions Spotter deals with are going to contain only analytic functions, then it makes no difference whether the random test values it uses for the variables are real or complex. Equality of analytic functions in a neighborhood along the real-number line implies equality off the line. However, this is not true if the expression contains non-analytic functions such as `abs`, `Re`, `Im`, `arg`, and `conj`. Therefore you may wish to use the `min_imag="..."` and `max_imag="..."` attributes to tell Spotter to test with non-real values.

In practice, the main issue is branch cuts. For instance, the student may provide the negative square root when you intended the positive one. You can handle this either by trying to anticipate all the possible forms the result could take, or, more cleanly, by supplying a `filter` attribute for the `ans` tag. For instance, suppose the student is asked to find the square root of x^2 . If you write the correct answer as `<ans e="x"/>`, then students who come up with $-x$ will be penalized for their creativity, and told that their answer is wrong. The solution is to write the answer tag as `<ans e="x" filter="abs(~)"/>`. The tilde, `~`, stands for the actual answers being compared (x and $-x$). The absolute value function is applied to these answers before they are compared for equality, so the student's answer of $-x$ will be recognized as a correct one.

By using filters, you can also deal with issues arising from the student's freedom to make choices. For instance, a physics problem may ask the student to predict the acceleration of an object moving in one dimension, without prescribing a coordinate system. Depending on the coordinate system the student chooses, this could come out either positive or negative. Another example is a problem in which we ask the student to compute a ratio, but the student has the freedom to define the ratio either way up. In this case, a `filter="abs(ln(~))"` does the trick. For angles, you can use `filter="~ mod 360"` or `filter="~ mod (2pi)"`.

If you've absorbed everything so far, then you're ready to understand how Spotter decides how many random sets of variables to use in its test for equality. The answer is that it uses only *one* set in the most commonly occurring case, where both the student's answer and the canned answer are analytic functions; otherwise it uses ten. The former case has essentially zero probability of giving an incorrect result. The latter, despite the larger number of tests, is the one that has a significant probability of messing up. For instance, an expression like $|x| + |y| + |z| + |x + y| + |y + z| + |x + z|$

slices up the (x, y, z) space into a crazy patchwork of regions, with analytic behavior only within each region. This is an unusual case, but I would still sleep better at night if I could increase the number of sample points to a hundred or a million. The problem is that the CGI is already annoyingly slow with ten points, so it's a trade-off. In the future, I plan to improve the efficiency of the code, which will allow a greater number of points to be used.

arbitrary multiplicative or additive constants

Sometimes an answer is only meaningful up to an over-all additive or multiplicative constant. An additive example would be an indefinite integral, which can have a constant of integration. A multiplicative example would be an expression that defines a proportionality, but we don't care about the constant of proportionality. These two cases can be handled by using `filter="-"` or `filter="/"`. The first tests whether the student's answer and the canned answer have a fixed difference. The second tests whether they have a fixed ratio.

numerical problems

The following is an example of an answer file entry for a numerical problem:

```

1    <problem id="two_plus_two">
2      <options unit_list="g,kg">
3        <find id="1">
4          2.00 kilograms plus 2.00 kilograms.
5          <ans e="4 kg" tol="0.01" tol_type="add"/>
6        </find>
7      </options>
8    </problem>

```

There are no variables. Line 2 specifies that a menu of units should appear to the right of the space in which the student types the number. She is expected to type in only the numerical part of the answer, since the units come from the menu. Line 5 specifies the answer with a tolerance of ± 0.01 kg. The `tol_type` attribute specifies that this tolerance is additive. If `tol_type` is omitted, the default is a multiplicative interpretation of the tolerance, ranging² from $x/(1+\epsilon)$ to $x(1+\epsilon)$. If the `tol` attribute is absent, the tolerance defaults to $\epsilon = 0.00001$ (the same as for symbolic problems). When specifying tolerances in scientific notation, use the notation `1e-4`, not Spotter's usual `10^-4`.

For the most part, there is no hard distinction between symbolic and numerical problems, and there is no qualitative difference in the way the software handles them. A numerical problem is simply one that happens to have no variables, and usually a wider range of tolerance than the default. Therefore it is not necessary to specify explicitly in the input file whether a problem is numerical or symbolic. As an exception, if you tell Spotter to check the number of significant figures on a problem, the implication is that the problem is numerical, and some qualitatively different behavior is triggered, as described in more detail below.

Sometimes we want the student to give an exact expression as an answer, e.g., $2/3$, rather than a decimal approximation such as 0.67 . In this situation, it's best to say explicitly in the descriptive text something like

²If the canned answer is real and the student's is complex, then the test is applied to their magnitudes, and a second test is imposed as well, that the argument of the student's answer must be less than both ϵ and 0.00001 . If both are complex, then the difference between their arguments must be less than ϵ .

“Enter an exact expression, not a decimal approximation.” Otherwise the student may enter 0.67 and not understand why it’s being marked wrong.

On the other hand, sometimes we *want* the student to enter the result as a decimal approximation, so that we can check the number of significant figures. For example, if the student is asked to compute the speed of an object that moves 2.00 meters in 3.00 seconds, we want them to enter 0.667, not $2/3$ or 0.6666667 (which is an error in significant figures). This is discussed in the following section.

checking significant figures

Spotter can check your students’ answers to see if they have the right number of significant figures. If the answer file gives the correct answer to a numerical problem with the optional `sig_figs` tag, then answers will be considered incorrect unless they are fully evaluated and have an appropriate number of sig figs. For example, suppose the correct answer is listed as `<ans e="4.0 kg" tol="0.3" tol_type="add" sig_figs="1-2"/>`. The `sig_figs="1-2"` indicates that the result should be given with 1 to 2 sig figs. If the student inputs 5 or 5.67 (in units of kg), the response is that the answer is incorrect. If the student inputs 4.10675432, the response is that the answer has an inappropriate number of significant figures. An input of $8/2$ would prompt a complaint that the answer had not been fully evaluated. Answers of 4, or 4.0, or 4.1 would all be considered correct.

The number of sig figs is given as a range. Usually what students do wrong is to give too many sig figs, implying false precision. There is usually not much point in setting the minimum number of sig figs to any value other than 0 or 1.

Spotter understands how to count sig figs in scientific notation, so the answer in the example above could be given as $4.0 \cdot 10^0$ or $4.0 \cdot 10^0$. Even though these inputs involve arithmetic operations such as exponentiation and (explicit or implied) multiplication, they are considered to have been fully evaluated. An answer such as 10^{12} is considered to have zero sig figs, and as a special case, 0 is also considered to have zero sig figs. For problems that are order-of-magnitude estimates, it can make sense to set `sig_figs="0-1"`, or even `sig_figs="0"`.

An input like 5500 is ambiguous; it could have 2, 3, or 4 sig figs. Spotter internally considers this to be 2 sig figs. Although some scientists (mostly chemists, it seems) use the presence or absence of a decimal point to eliminate this ambiguity, this convention is not universally followed or understood, so Spotter’s behavior in such a situation is undefined, and you should not count on its behaving in a particular way.

Sometimes a problem will require that the student look up data in a book or online — e.g., the mass of the electron or the speed of sound. For this reason, it is not possible for Spotter to check a student’s sig figs as thoroughly as a human reading the entire solution. It doesn’t know how many sig figs were in the data the student looked up, so it doesn’t know how many should be in the student’s result. If the student looks up a value of the data that has inappropriately low precision (e.g., $g=10 \text{ m/s}^2$ in a 3-sig-fig problem), then presumably the answer will be incorrect simply because it doesn’t fall within the range of tolerance set in the answer file.

debugging your answer file

There are two types of mistakes you can make in your answer file: mistakes in the answers, and mistakes in the format.

Suggestions for catching and avoiding mistakes in the answers:

1. For symbolic problems, run Spotter, and input an answer that is incorrect, but has the right units. For instance, if the problem calls for calculating work, input 1 J (1 joule) as the answer. (Note that although students are not ever supposed to have to type in units, the software will accept such an answer to a symbolic question.) If Spotter responds simply by saying the answer is incorrect, then you've verified that the answer you put in has the right units. If it says your answer has the wrong units, then your symbolic answer is incorrect.

2. For numerical problems, write the canned answer as an expression, rather than as the numerical result of the expression. Then test it using the numerical result of the expression. (You can also do it the other way around, and that will improve the program's performance a little. However, if you come back to the answer file later, it is more obvious that the expression form is correct.)

I've worked hard to make Spotter give helpful error messages to students. The same can't be said for the way it handles errors in your answer file, so catching and avoiding mistakes in your format isn't always as easy as it should be. The error messages may go in your log file if you're lucky, but if you're not, they'll end up in the Apache error log, which is usually hard to get access to. Suggestions:

1. Start simple, and test early and often.

2. Use a validating XML parser to check your format against the file `spotter.dtd`. There are web-based services that can do this for you, which typically requires that you cut and paste the contents of the DTD file into your answer file as its second line. To do this on linux from the command line, you can use the command `xmllint --noout --dtdvalid spotter.dtd answers.xml`. The utility `xmllint` is packaged for debian-based systems as part of `libxml2-utils`.

6 Qualitative Questions

Although I originally designed Spotter to check answers to quantitative questions, it can also do qualitative ones. The mechanism for this is very general. You can do simple multiple-choice questions, but there is also an interface to JavaScript so that you can write questions in which the students is effectively interacting with a complex computer program. The following is an example of a multiple-choice question.

```
1 <problem id="yoko" type="mc">
2   <find id="1">
3     Which Beatle was married to Yoko Ono?
4     <data array="mc">
5       "John",",
6       "Paul","Paul didn't even like Yoko.",
7       "George","George didn't even like Yoko.",
8       "Ringo","Ringo didn't even like Yoko."
9     </data>
10  </find>
11 </problem>
```

Line 3 is the question, and lines 5-8 are the choices. After each choice, there is either a null string (for the correct answer) or a string explaining why the answer is wrong (for the rest of them). When the student clicks on John in this example, he's told his answer is correct. If he clicks on one of the other answers, he's told why that answer is incorrect, and is allowed to try again until he hits the right one.

Unlike Spotter's facilities for quantitative questions, the ones for qualitative questions don't try to keep any secrets from the student, and therefore should not be used for high-stakes grading. Correct answers are recorded, but with the type of multiple-choice problem shown above, the student can always end up getting the right answer simply by clicking on all the choices. Even for other types of qualitative questions, it's trivial for the student to, e.g., do View Source in the browser, and see all the information in the problem, before he clicks on anything. The mechanism used for reporting correct answers to qualitative questions uses a fancy feature of modern web browsers (called XMLHttpRequest) so that the student doesn't have to click on a button to submit his response; this should work in recent versions of Firefox, and Internet Explorer 6 or higher.

If you just want to use multiple-choice problems, you don't have to know anything about JavaScript. However, in general, the interface to JavaScript works like this. The `type` parameter corresponds to a JavaScript file, in this case `mc.js`, which exists in the `spotter_js` subdirectory of the server's data directory. The string in the `type` parameter is also used for several other purposes. Inside the JavaScript file, there should be a function called `populate`. A `data` tag in the XML file should be used to create a JavaScript variable (typically an array) whose name is (in our example) `mc`. This variable will automatically be passed to the function `populate`, whose job is to initialize the HTML stuff displayed on the screen; that stuff all goes inside an HTML `div` block whose id is `container`.

7 Abuse

My experience is that most students use Spotter in a positive way, to improve their education, but a few will do silly things like random guessing. Spotter has a built-in throttling mechanism to prevent a user from trying too many answers to the same problem in a short period of time. The student can enter no more than 1 answer to the same problem in any interval of 30 seconds, and also no more than 2 in 3 minutes, 3 in 15 minutes, 4 in one hour, and 5 in one day. This is implemented by logging the use of the program in daily log files in the `spotter/throttle` directory. (The directory is automatically created if you don't create it yourself.) The old log files can be deleted. Both anonymous and logged-in users are tracked in these files. In the case of logged-in users, more detailed information is recorded in the individual student's `.work` file (see below), and it's sometimes interesting to browse through these and see what the students have been doing.

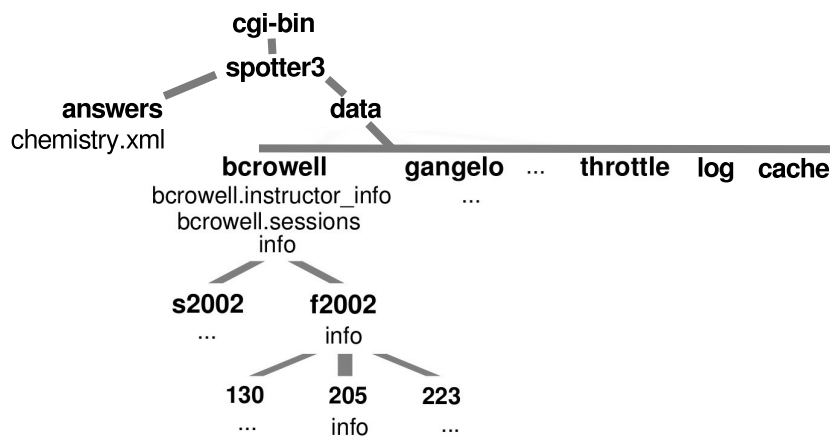
To disable throttling, create a file called `exempt_files` in the directory `spotter3/data/throttle`. If you want throttling to be turned off when people the the answer file `foo.xml`, enter `foo.xml` on a line by itself in `exempt_files`.

One way to get around the throttling mechanism is to log out and move to

a different computer. By doing this, one can enter twice as many guesses. To discourage this, you can forbid anonymous use from computers on your own campus. Make a file in the `spotter/throttle` directory called `forbid_anonymous_use_at`, and put in it, on a line by itself, the beginning of your school's IP addresses. For instance, IP addresses on my campus all begin with 207.233, which I found out by doing `dig fullcoll.edu` on a Unix machine.

8 Interfacing

If Spotter is only being used to allow students to check their answers, the setup on the server is simple: the software goes in the server's `cgi-bin` directory, along with the XML file containing the answers. Things get more complicated, however, if you want to record their answers, in which case you need to set up a directory tree like the one in the figure. In this diagram, bold-face indicates the name of a directory, which contains files shown in ordinary type, and subdirectories represented as sub-branches in the tree (which is shown upside-down, as is customary in computer science). You also need to set up this kind of directory tree if you want to use Spotter to give students access to their grade reports via the web. My OpenGrade grade-recording software is designed to interface with Spotter through this kind of directory tree. From the OpenGrade client software, you can post grade reports to the server, and you can also download information about the students' answers. You only need to create the `cgi-bin/spotter3` directory manually. Everything below that level can be set up using the `admin_spotter` script, as described later in this section.



cgi-bin — This is the directory on your server that houses CGI software: programs that generate customized web pages on the fly.

ochem.xml — Your answer file.

spotter — To keep Spotter's files from cluttering up the `cgi-bin`, nearly all of them are contained within this subdirectory. It should be owned by the group that Apache runs under (`www` on FreeBSD, possibly `apache` on some other systems). The users (teachers) should all be members of this group.

bcrowell — A directory containing a particular instructor's classes.

`bcrowell.instructor_info` — Stores the instructor's passwords in encrypted

form. There can be more than one of these files, e.g., to allow access to teaching assistants. Example:

```
name="Mr. Science" email="askmrs@heknowsmorethanyoudo.com"
server_key="98676775467956759867"
password_hash="C+7Hteo/D9vJXQ3UfzxbwnXaijM"
```

The server key is a long, random, password that would be impractical to memorize, but that provides good security; it is stored permanently on the server and client computers. The password hash is the encrypted form of a memorable password chosen by the user, which needs to be typed in every time the instructor's client software connects to the server; normally this password will be the same as the password the instructor uses for tamperproofing OpenGrade's gradebook files. (The encryption is defined by `SHA1("spotter_instructor_password",password)`.) An unethical student who gets access to the instructor's account on the client machine will be able to find out the server key, but will not be able to access the server without knowing the password encrypted in the password hash. If the instructor chooses a bad password that is vulnerable to dictionary attacks, it won't matter unless the hacker can also lay hands on the server key.

The name and e-mail are only used for displaying the instructor's e-mail address along with the students' e-mail directory. As with students' e-mails, this is not accessible to anyone who's not logged in to a valid Spotter account. If you don't want your e-mail listed in the directory, you can leave out the name and e-mail data completely.

`bcrowell.sessions` — A security feature; provides protection against a certain class of attacks known as "replay" attacks. Technical details: every time an instructor does something on the server, a unique session identifier is appended to this file. If someone has intercepted the packets and tries to reuse them, this will be detected.

`info` — Some data relating to the instructor: a description, a flag that allows the instructor's account to be disabled, and an expiration date for the account. Example:

```
description="Ben Crowell's classes at Fullerton College"
disable="0"
expire=""
```

`f2002` — The files for a particular semester.

`info` — Similar to the instructor's info file.

`205` — A directory for a particular class.

`info` — Similar to the instructor's info file.

`dates` — Allows you to control the time period during which students can use Spotter for certain problems. This is not yet implemented, and should seldom be necessary, because you can just ignore work a student has done after due date.

`journals` — You create this file if you want your students to be able to maintain a journal in Spotter. Example:

```
"your diary","diary"
```

Logged-in students will be presented with an option "edit your diary," and Vijay Patel's diary file will be `patel_vijay.diary`. As a shortcut, you can simply use a line like this:

"your diary"

and then the file will be called `patel_vijay.yourdiary`.

`newton_ike.grade_report` — A grade report for student Ike Newton, in HTML format. OpenGrade generates these reports and uploads them.

`newton_ike.info` — Contains information about the student's account. Example:

```
password="vmpTsU+vjzfIIqYgORq15KUpSG8",last="Newton",first="Ike",disabled="0"
state="normal",email="ike@oxbridge.edu",emailpublic="1",newpasswordkey=""
id="123456"
```

The password field contains the student's password, in encrypted form; emailpublic indicates whether the student has allowed other students in the class to see his e-mail address; disabled should be set to 1 if the student drops the class; id is his student id number. When the student's account is first set up, it looks like this:

```
last="Patel",first="Vijay",id="00640121",disabled="0"
state="notactivated",password="k+23qL+pTsYvvRPZ1IZxeSBaqw8"
```

His password is the same as his student id, and the password field contains that password, encrypted in the usual way. The state is set to notactivated. When the student logs in for the first time, he must activate his account by supplying a real password, and he will also be allowed to enter an e-mail address, and choose whether he wants it to be visible to the other students.

`newton_ike.work` — A log of the answers that Ike has inputted into Spotter while logged in. This file doesn't have to be created as part of the initial setup; Spotter just creates it automatically.

`newton_ike.old_work` — After answers have been downloaded and examined using OpenGrade, they are removed from the work file and put into the old_work file (not yet implemented).

messages — This optional directory holds announcements you've made to the whole class, or to particular students or sets of students. It will be created the first time you use OpenGrade to post an announcement.

`2003-02-17-093711-jzGf` — This is a message that you posted on Feb. 17, 2003, at 9:37:11. (The final four letters are a hash computed from the message's contents, intended merely to make sure that every filename is unique.) Example:

```
subject=final exam canceled
```

Since everyone in the class is doing so well, I've decided to cancel the final.

Best wishes to everyone for an enjoyable summer vacation!

The file starts with one or more header lines, of which the only mandatory one is the subject line. Header lines that are not understood are discarded. After that comes a blank line, and then the body of the message, with blank lines separating paragraphs.

`newton_ike` — This file inside the messages directory contains information about the messages that have been sent to Ike Newton. Example:

```
sent,2003-02-17-093713,2003-02-17-093711-jzGf
```

received,2003-02-18-170155,2003-02-17-093711-jzGf

Only one message has been sent to Ike Newton. On Feb. 18, at 5:01:55 pm, he logged in to Spotter and was presented with the message.

In addition to the files discussed above, there are some other files, which have “floating” locations, the only current examples being banner.html and footer.html, which contain HTML code that Spotter inserts at the top and bottom of the web pages it generates. Spotter expects to find these files somewhere in the tree, but is flexible about where. It starts by looking for a floating file in the class’s directory, but if it doesn’t find it there, it looks in the parent directory, then its parent, etc., going all the way up to the `spotter` directory if necessary. For example, user bcrowell could have a banner.html file in his bcrowell directory that would provide a link to his own web page, and gangelo could have her own version in her own directory.

setting up Here’s a summary of what you have to do to set up the file tree from scratch. First, create the `cgi-bin/spotter3` directory if it doesn’t already exist, and do `chmod ug+rw` on it. Do the same for the subdirectory `cgi-bin/spotter3/data`.

Make sure that the Apache web server (typically user `www` or `www-data`) is in the group that owns the directories, so that it can read and write the files in them. Add yourself to this group by editing `/etc/group`.

Run the `admin_spotter` script, and use the “ai” command to create the instructors’ directories and info files.

Create a plain text file that has a line like this for each student:

```
123456 Smith, John
```

This can be created by hand, or, at Fullerton College, by using the script `admin_spotter` script and choosing the “fc” menu option. It can be helpful to add a fake student to this file for testing purposes.

In the `admin_spotter` script, use the “e” menu option to create your classes, and then use “i” to import the roster. (If you use OpenGrade, it also makes a file `og_roster_section` that can be pasted into an OpenGrade file.)

The students will be getting into Spotter from the class’s web page, which should have a link that looks something like this:

Click

```
<a href="cgi-bin/spotter3/Spotter.cgi?file=lm&login=form&what=check&class=bcrowell/s2003/206">
```

```
here</a> to check your answers with Spotter or to check your current grade in the class.
```

If you’re testing this on your desktop machine, using the sample answer file, the url would be something like this:

```
http://localhost/cgi-bin/spotter3/Spotter.cgi?what=check&file=sample&login=form&class=bcrowell/s2003/206
```

If you have users who don’t know anything about Linux, the easiest way to give them control over their classes is by making the `admin_spotter` script be their login shell. They can ssh to the server, enter their password, and be instantly put into the script’s main menu.

9 Journals

I use “journals” as a generic term for text files that students can edit and maintain online through Spotter. You can annotate your students’ journals. For instance, in one of my classes I have my students hand in their lab reports electronically using Spotter’s journal mechanism. To set up journaling, you have to create a `journals` file as described in the preceding section.

Students edit their journals in a simple markup language, which is then displayed in their web browser as formatted text. The following is an example of some markup:

```
=Planets
==Jupiter
I observed Jupiter this week with binoculars, and
saw some of its moons.
*Jan. 23      saw two moons like this:      .  0 .
*Jan. 25      they moved!                    .  0 .
*Jan. 26      a third one appeared          .  .0 .

This was a really [[cool//Sally, I'm glad you had
fun. Nice job! -Prof. Longhair]]
project, and I'm glad we did it.
```

The `=` and `==` symbols at the beginning of a line make section and subsection headings. Lines that begin with an asterisk are formatted exactly as they were typed. A blank line divides two paragraphs. The notation `[[//]]` is for an annotation added by the instructor. In this example the word “cool,” written by the student, will be underlined, and the comment will appear in the margin next to the student’s paragraph. Annotations can also be written without the double slash, in which case everything inside the double brackets is taken to be a comment.

Annotating journals is currently pretty primitive; you simply have to open the student’s journal file in a text editor. Usually a journal will be due on a certain date. On that date, you can lock the journal (prevent further editing by the student) by creating a file with the extension `.lock`. For example, if the journal file is called `jones_sally.observing`, you would create a file called `jones_sally.observing.lock`.

10 Bugs

known bugs in the parser The following expression was putting the lexer into an infinite loop:

```
(mcos theta)^2+((mcos)(muk)+bv^2)^2
```

Here `m`, `theta`, `muk`, and `b` were defined variables, and `v` was not a defined variable. I fixed the infinite loop simply by putting a maximum limit on the number of recursions the lexer can do, but this obviously shows an underlying logical mistake. Haven’t had any luck yet figuring this out. Simplifying the expression in any way seems to eliminate the problem. The *depth* of the recursion isn’t infinite. One possibility is that it isn’t really an infinite loop but simply an expression that takes a very long time to lex using my algorithm. If so, I should figure out why this particular

expression is so bad.

An expression like 1.2.3 is uncomplainingly parsed as $(1.2)(.3)=0.36$. Spotter should complain about the ambiguity.

other things to improve in the parser

The selection of units is fixed: the source code has the hardcoded symbol `\%Spotter::standard_units` sprinkled everywhere. Also `%Spotter::accepts_metric_prefixes` is all over the place. I think I need to redesign this whole aspect of the software to be object-oriented, so that the set of symbols (units, prefixes, constants, functions, variables) is an object. It needs to interface to the Expression class, and it needs to be tested with the test suite, so the calculator needs to be rewritten to use the Expression class. A related ugliness is where I have to do `delete($Spotter::standard_cons_hash{$sym});` in `Spotter.cgi`.

Contrary to the documentation, an expression like $1/2x$ is interpreted as $1/(2x)$. Is this a bug, or a feature? Anyway, it should give a warning here.

The `mod` operator accepts complex numbers, but I'm not convinced that the way it handles them is the best definition mathematically.

An expression like $1/12 \text{ ft}$ should produce a warning.

An expression like $2 + +2$ is evaluated as 4.

I intentionally haven't included comparison operators other than `eq`, since all operators are supposed to work equally well for real and complex numbers. However, it would be useful to be able to specify a range of numbers, and test whether a number lay in that range, e.g. `x in box(a,b)` would test whether `x` lies in the box in the complex plane whose corners are `a` and `b`. This would require implementing functions with multiple arguments (probably would work already – just not tested). The evaluator would have to recognize a new kind of object, `Set`, representing a set of numbers in the complex plane. There could be other operators besides `box` for creating `Set` objects, e.g. you could make infinite and semi-infinite strips, disks, ... I don't think this would be terribly difficult to do, as long as I didn't try to implement set operations like unions, etc. Boxes should extend a little bit outside their specified borders, so that e.g. a zero-height box lying on the real axis will include numbers that have a slight rounding error in their near-zero imaginary part. Until this is implemented, it's possible to work around sometimes using filters, e.g. a negative answer can be checked for using `filter="arg(^)"`.

known bugs in the CGI

There seems to be a problem when a student requests an e-mail to reset his password, and then does it again without waiting to get the first e-mail. The second request doesn't seem to change the secret key in the student's info file, so the link it gives is nonfunctional.

Journals, work files, etc. should all be created with mode `g+w`.

`Spotter.cgi` doesn't have `-wT` on the bang line, and it breaks if you add it.

The code that truncates long problem descriptions may chop `<` or html tags apart.

The program crashes if the URL refers to a problem that isn't in the answer file.

I think there may be some bugs lurking where I copy references rather than cloning objects. I fixed a bug like this in the `options_stack_dup`

other things to improve in the CGI	code, but there may be more. Should write a clone method into each class.
	When the canned answer contains an error like subtracting units that aren't commensurate, it prints the canned answer, plus a lot of debugging output.
	It should die with a fatal error when two <code>find</code> tags in the same problem have the same id.
	When the answer file isn't well-formed, Spotter crashes with an error message in the log file.
	It should be possible to do tolerances and answers in the same kind of scientific notation.
known bugs in the calculator	The calculator allows you to do assignments like <code>1=2</code> .

11 Acknowledgements

The idea for Spotter is based on a numerical answer checker called Capa developed at Michigan State University. Capa is also open source, and recent versions of Capa can do symbolic answers.

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